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Smooth point-set registration using neighboring constraints

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ABSTRACT

We present an approach for Maximum Likelihood estimation of correspondence and alignment parameters that benefits from the representational skills of graphs. We pose the problem as one of mixture modeling within the framework of the Expectation–Maximization algorithm. Our mixture model encompasses a Gaussian density to model the point-position errors and a Bernoulli density to model the structural errors. The Gaussian density components are parameterized by the alignment parameters which constrain their means to move according to a similarity transformation model. The Bernoulli density components are parameterized by the continuous correspondence indicators which are updated within an annealing procedure using Softassign. Outlier rejection is modeled as a gradual assignment to the null node. We highlight the analogies of our method to some existing methods.

We investigate the benefits of using structural and geometrical information by presenting results of the full rigid version of our method together with its pure geometrical and its pure structural versions. We compare our method to other point-set registration methods as well as to other graph matching methods which incorporate geometric information. We also present a non-rigid version of our method and compare to state-of-the-art non-rigid registration methods.

Results show that our method gets either the best performance or similar performance than state-of-the-art methods.

1. Introduction

Alignment of point-sets is frequently used in *pattern recognition* when objects are represented by sets of coordinate points. The idea behind is to be able to compare two objects regardless the effects of a given transformation model on their coordinate data. This is at the core of many object recognition applications where the objects are defined by coordinate data (e.g., medical image analysis, shape retrieval, ...), learning shape models (Dryden and Mardia, 1998; Cootes et al., 1995) or reconstructing a scene from various views (Hartley and Zisserman, 2000).

Given that the correspondences are known, there is an extensive work done towards the goal of finding the alignment parameters that minimize some error measure. To cite a few, Dryden and Mardia (1998) and Kendall (1984) deal with isometries and similarity transformations; Berge (2006), and Umeyama (1991) deals with Euclidean transformations (i.e. excluding reflections from isometries); Haralick et al. (1989) deal with similarity and projective transformations; and Hartley and Zisserman (2000) deal exclusively with projective transformations.

However, the point-set alignment problem is often found in the more realistic setting of unknown point-to-point correspondences. This problem becomes then a *registration problem*, this is, one of jointly estimating the alignment and correspondence parameters. Although non-iterative algorithms exist for specific types of transformation models (Ho and Yang, 2011), this problem is usually solved by means of non-linear iterative methods that, at each iteration, estimate correspondence and alignment parameters. Despite being more computationally demanding, iterative methods are more appealing to us than the direct ones due to its superior tolerance to noise and outliers.

We distinguish between two families of approaches at solving this problem. Ones are based on the *Expectation–Maximization* (EM) algorithm (Dempster et al., 1977), and the others use *Softassign* (Gold and Rangarajan, 1996; Gold et al., 1998; Rangarajan et al., 1997). The former ones have the advantage of offering statistical insights of such decoupled estimation processes while the latter ones benefit from the well-known robustness and convergence properties of the Softassign embedded within deterministic annealing procedures.

Myronenko and Song (2010) proposed *Coherent Point Drift* (CPD), a point-set registration method using the EM algorithm that is defined for rigid, affine and non-rigid transformations. Gold et al. (1998), Rangarajan et al. (1997) proposed *Robust Point Matching*

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(RPM), a method using Softassign that is defined for affine and rigid transformations. Later, Chui and Rangarajan (2003, 2000) presented *TPS-RPM*, its extension to non-rigid transformations.

Graph matching approaches allow for neighboring relations between points into the point-set registration problem. *Graduated Assignment* by Gold and Rangarajan (1996) is a remarkable graph matching method using Softassign. Cross and Hancock (1998) presented an approach for graph matching and point-set alignment using the EM algorithm that was defined for affinities and projectivities. One limitation of this approach is the high computational demand of the dictionary-based structural model. Luo and Hancock (2003) proposed an EM-like approach for graph matching and point-set alignment based on a cross-entropy measure. They proposed a model of structural errors based on a Bernoulli distribution. This model was defined for rigid-body transformations.

We propose a joint structural graph matching and point-set registration method whose main contributions are the following:

- We try to bridge the gap between the EM-based and the Softassign-based approaches by formulating the graph matching problem within a principled statistical framework, while benefiting from the desirable properties of the Softassign and deterministic annealing ensemble.
- Correspondence problem is approximated as a succession of linear assignment problems which are solved using Softassign. This way, we are able to use continuous correspondence variables as opposed to other approaches that use discrete ones (Cross and Hancock, 1998; Luo and Hancock, 2003).
- Outlier rejection is modeled as a smooth assignment to the null node within the annealing procedure.
- The proposed model can be easily adapted to allow only for geometric or structural information. We show how it can be seen as a more general framework with clear connections to other well-known methods.
- Although the proposed method deals with rigid transformations we show how it can be embedded into a non-rigid deformation procedure thus obtaining similar or better performances than state-of-the-art non-rigid registration methods.

The outline of this paper is the following. In Section 2 we formulate the matching problem as one of mixture modeling with missing data and propose our mixture model. In Section 3 we derive the EM algorithm for our model. Section 4 presents the methodology used to reject outliers. In Section 5 we highlight parallelisms of the proposed method with some other existing methods. In Section 6 we present some experiments and results, and finally in Section 7 conclusions are given.

2. A mixture model

Consider two graph representations $\mathcal{G} = (\mathcal{U}, D, \mathcal{X})$ and $\mathcal{H} = (\mathcal{V}, M, \mathcal{Y})$ extracted from two images.

The node-sets $\mathcal{U} = \{u_a,\ a \in \mathcal{I}\}$ and $\mathcal{V} = \{v_\alpha,\ \alpha \in \mathcal{J}\}$ contain the symbolic representations of the nodes, where $\mathcal{I} = 1, \ldots, |\mathcal{U}|$ and $\mathcal{J} = 1, \ldots, |\mathcal{V}|$ are their index-sets.

The vector-sets $\mathcal{X} = \{\mathbf{x}_a,\ a \in \mathcal{I}\}$ and $\mathcal{Y} = \{\mathbf{y}_\alpha,\ \alpha \in \mathcal{J}\}$, contain the column vectors $\mathbf{x}_a = (x_a^V, x_a^H)^\top$ and $\mathbf{y}_\alpha = (y_\alpha^V, y_\alpha^H)^\top$ of the two-dimensional coordinates (vertical and horizontal) of each node, where $\top$ denotes the transpose operator.

The adjacency matrices D and M contain the edge-sets, encoding some kind of relation between pairs of nodes (e.g., connectivity or spatial proximity). Hence, $D_{ab} = \begin{cases} 1 & \text{if } u_a \text{ and } u_b \text{ are linked by an edge} \\ 0 & \text{otherwise} \end{cases}$ (the same applies for $M_{\alpha\beta}$).

We use continuous correspondence indicators S so, we denote as $s_{a\alpha} \in S$, the probability of node $u_a \in \mathcal{U}$ being in correspondence with node $v_\alpha \in \mathcal{V}$.

It is satisfied that

$$\sum_{\alpha \in \mathcal{J}} s_{a\alpha} \leqslant 1, \quad a \in \mathcal{I} \tag{1}$$

where, $1 - \sum_\alpha s_{a\alpha}$ is the probability of node u_a being an outlier.

Our aim is to recover the correspondence indicators S and the alignment parameters Φ that maximize the *observed-data* likelihood of the data-graph $P(\mathcal{G}|S, \Phi)$. Within this setting, constraints on the data-graph $\mathcal{G}$ are evaluated on the model-graph $\mathcal{H}$. To make this problem tractable, we introduce the *hidden variables*, namely, the corresponding model graph nodes $v_\alpha \in \mathcal{V}$.

By assuming that the observations are independent and identically distributed, the observed-data likelihood writes

$$P(\mathcal{G}|S, \Phi) = \prod_{a \in \mathcal{I}} \sum_{\alpha \in \mathcal{J}} P(u_a, v_\alpha | S, \Phi) \tag{2}$$

Following a similar development than (Luo and Hancock, 2001) we factorize, using the Bayes rules, the *complete-data* likelihood in the right hand side of Eq. (2) into terms depending on individual correspondence indicators, in the following way.

$$P(u_a, v_\alpha | S, \Phi) = K_{a\alpha} \prod_{b \in \mathcal{I}} \prod_{\beta \in \mathcal{J}} P(u_a, v_\alpha | s_{b\beta}, \Phi) \tag{3}$$

where $K_{a\alpha} = [1/P(u_a|v_\alpha, \Phi)]^{|\mathcal{I}| \times |\mathcal{J}| - 1}$. If we assume that conditional dependence of data-graph node u_a can only be taken into account in the presence of the correspondence matches S, then $P(u_a|v_\alpha, \Phi) = P(u_a)$. Further assuming equiprobable priors $P(u_a)$, we can safely discard these quantities in the maximization of Eq. (2), since they do not depend either on S nor Φ.

We propose a measure for the complete-data likelihood of Eq. (3) that combines a model of structural errors based on a Bernoulli distribution augmented with a model of geometric errors based on a Gaussian distribution.

With regards to the structural relations, Luo and Hancock (2001) proposed to model the likelihood of an observed relation given the hypothesis on the correspondences using a Bernoulli distribution with parameters S. This is, given two corresponding pairs of nodes $u_a,\ u_b \in \mathcal{U}$ and $v_\alpha,\ v_\beta \in \mathcal{V}$, they assumed that there will be edge-discordance (i.e., $D_{ab} = 0 \vee M_{\alpha\beta} = 0$) with a fixed (low) probability of error P_e. Otherwise, there will be edge-concordance with probability $1 - P_e$. This is,

$$P(u_a, v_\alpha | s_{b\beta}) = \begin{cases} (1 - P_e) & \text{if } D_{ab} = 1 \wedge M_{\alpha\beta} = 1 \wedge s_{b\beta} = 1 \\ P_e & \text{otherwise} \end{cases} \tag{4}$$

With regards to the geometrical measurements, it is reasonable to consider that point-position errors between corresponding points follow a Gaussian density. In the case of no correspondence, we use a fixed probability ρ that will model the outlier process. This is,

$$P(u_b | s_{b\beta}, \Phi) = \begin{cases} P_{b\beta}^{(\Phi)} & \text{if } s_{b\beta} = 1 \\ \rho & \text{otherwise} \end{cases} \tag{5}$$

where $P_{b\beta}^{(\Phi)}$ is a Gaussian measurement on the point-position errors with parameters Φ. This is,

$$P_{b\beta}^{(\Phi)} = \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\|\mathbf{x}_b - \mathcal{T}(\mathbf{y}_\beta; \Phi)\|_\Sigma^2\right] \tag{6}$$

where $\mathcal{T}(\mathbf{y}_\beta; \Phi)$ represents the geometric transformation of model point $\mathbf{y}_\beta$ according to alignment parameters Φ, and $\|\mathbf{d}\|_\Sigma^2 = \mathbf{d}^\top \Sigma^{-1} \mathbf{d}$