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Fast and low-complexity method for exact computation of 3D Legendre moments

Khalid M. Hosny *

Department of Information Technology, Faculty of Computers and Informatics, Zagazig University, Zagazig, Egypt

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ABSTRACT

A new method is proposed for fast and low-complexity computation of exact 3D Legendre moments. The proposed method consists of three main steps. In the first step, the symmetry property is employed where the computational complexity is reduced by 87%. In the second step, exact values of 3D Legendre moments are obtained by mathematically integrating the Legendre polynomials over digital image voxels. An algorithm is employed to significantly accelerate the computational process. In this algorithm, the equations of 3D Legendre moments are treated in a separated form. The proposed method is applied to determine translation-scale invariance of 3D Legendre moments in a very simple way. Numerical experiments are performed where the results are compared with those of the existing methods. Complexity analysis and results of the numerical experiments clearly ensure the efficiency of the proposed method.

1. Introduction

Image analysis using the orthogonal Legendre moments was introduced by Teague (1980). Legendre orthogonal moments can be used to represent an image with the minimum amount of information redundancy (Teh and Chin, 1988). Based on these attractive properties, Legendre moments are used in many applications such as pattern recognition (Chong et al., 2004, Luo and Lin, 2007), face recognition (Haddadnia et al., 2001), line fitting (Qjidaa and Radouane, 1999), texture analysis (Bharathi and Ganesan, 2008), template matching (Omachi and Omachi, 2007, Hosny, 2010b), palm-print verification (Pang et al., 2003), occupant classification system for automotive airbag suppression (Farmer and Jain, 2003), comparison of two-dimensional polyacrylamide gel electrophoresis maps images (Marengoa et al., 2005), retrieval and classification (Yadav et al., 2008), and tool wear monitoring (Barreiro et al., 2008). It is well known that, the direct computation of Legendre moments is time consuming process and the computational complexity increased by increasing the moment order.

Remarkable methods have been proposed to improve the accuracy and reduce the computational times of Legendre moments (Yap and Paramesran, 2005; Hosny, 2007a,b). These methods are devoted to computing 2D Legendre moments. In the literature of moment computation, only one work is presented to compute 3D Legendre moments. Hsu et al. (2001) proposed a method to approximately compute Legendre moment for polyhedra. They applied Gaussian theorem to transform the volume integral into a surface one, then deduced the double integral from the simple integral by a Green's theorem. As we know, polyhedra are special cases of 3D objects. This is the main weak points of their work. Hsu and his co-authors consider binary images only. They assume that, the 3D binary images/objects are entirely determined by their boundary surface. This is another weak point.

Recently, 3D images gains more interest where representation and description of 3D features and the reconstruction of 3D image/objects are very important in many scientific fields such as medical imaging, multimedia and molecular biology. The lack of fast and accurate 3D Legendre moments and moment invariants is the motivation of this study.

This paper proposes a new method for fast and low-complexity computation of exact 3D Legendre moments. Symmetry property is employed where 87% of the computational complexity is reduced. The set of 3D Legendre moments are computed exactly by using a mathematical integration of Legendre polynomials over the object voxels. Then, a fast algorithm is applied in order to accelerate the computational process. Translation-scale 3D Legendre moment invariants are derived in a very simple way. Detailed complexity analysis of the proposed and conventional methods is presented. Complexity analysis and the results of conducted numerical experiments clearly show the efficiency of the proposed method.

The rest of the paper is organized as follows: in Section 2, an overview of 3D Legendre moments and object reconstructions are given. The proposed method is described in Section 3. Section 4 is devoted to discussing the computational complexity and the results of numerical experiments. Conclusion and concluding remarks are presented in Section 5.

* Current address: Department of Computer Science, Najran Community College, Najran University, P.O. Box 1988, Najran, Saudi Arabia. Tel.: +966 055 3517581; fax: +966 07 5440357.

E-mail address: k_hosny@yahoo.com

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2. 3D Legendre moments

The 3D Legendre moments of order $(p + q + r)$ for an image/object with intensity function $f(x,y,z)$ are defined over the cube $[-1,1] \times [-1,1] \times [-1,1]$ as:

$$L_{pqr} = \lambda_{pqr} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} P_p(x) P_q(y) P_r(z) f(x,y,z) dxdydz, \tag{1}$$

where the coefficient λ_{pqr} is defined as:

$$\lambda_{pqr} = \frac{(2p+1)(2q+1)(2r+1)}{8}. \tag{2}$$

Legendre polynomial, $P_p(x)$, of order p is defined as:

$$P_p(x) = \sum_{k=0}^{p} B_{k,p} x^k \tag{3}$$

with $x \in [-1,1]$ and the coefficient matrix $B_{k,p}$ defined as:

$$B_{k,p} = (-1)^{\left(\frac{p-k}{2}\right)} \frac{1}{2^p} \frac{(p+k)!}{\left(\frac{p-k}{2}\right)!\left(\frac{p+k}{2}\right)!k!}. \tag{4}$$

The Legendre polynomial, $P_p(x)$, obeys the following recursive relation:

$$P_{p+1}(x) = \frac{(2p+1)}{(p+1)} x P_p(x) - \frac{p}{(p+1)} P_{p-1}(x) \tag{5}$$

with $P_0(x) = 1$, $P_1(x) = x$ and $p > 1$. The set of Legendre polynomials $P_p(x)$ forms a complete orthogonal basis set on the interval $[-1,1]$. The orthogonality property is defined as:

$$\int_{-1}^{1} P_p(x) P_q(x) dx = \begin{cases} 0, & p \neq q, \\ \frac{2}{(2p+1)}, & p = q. \end{cases} \tag{6}$$

A digital 3D image or object of size $M \times N \times K$ is a multidimensional array of voxels. Centers of these voxels are the points (x_i,y_j,z_k), where the image/object intensity function is defined only for this discrete set of points $(x_i,y_j,z_k) \in [-1,1] \times [-1,1] \times [-1,1]$. The sampling intervals in the x -, y- and z-directions $\Delta x_i = x_{i+1} - x_i$, $\Delta y_j = y_{j+1} - y_j$, $\Delta z_k = z_{k+1} - z_k$, respectively. In the literature of digital image processing, the sampling intervals Δx_i, Δy_j and Δz_k are fixed at constant values $\Delta x_i = 2/M$, $\Delta y_j = 2/N$ and $\Delta z_k = 2/K$ respectively. Therefore, the points (x_i,y_j,z_k) will be defined as follows:

$$x_i = -1 + \left(i - \frac{1}{2}\right)\Delta x, \quad y_j = -1 + \left(j - \frac{1}{2}\right)\Delta y \quad \text{and}$$
$$z_k = -1 + \left(k - \frac{1}{2}\right)\Delta z \tag{7}$$

with $i = 1,2,3,\ldots,M$, $j = 1,2,3,\ldots,N$ and $k = 1,2,3,\ldots,K$. For the discrete-space version of the image, Eq. (1) is usually approximated by:

$$\widetilde{L}_{pqr} = \frac{(2p+1)(2q+1)(2r+1)}{MNK} \times \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{K} P_p(x_i) P_q(y_j) P_r(z_k) f(x_i,y_j,z_k). \tag{8}$$

Moments calculated using Eq. (8) are the approximated zeroth-order (ZOA) 3D Legendre moments.

2.1. Object reconstruction using 3D Legendre moments

Since, Legendre polynomial $P_p(x)$ forms a complete orthogonal basis set on the interval $[-1,1]$ and obeys the orthogonal property, the image/object intensity function $f(x,y,z)$ can be expressed as an infinite series expansion in terms of the Legendre polynomials over the cube $[-1,1] \times [-1,1] \times [-1,1]$:

$$f(x,y,z) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} L_{pqr} P_p(x) P_q(y) P_r(z), \tag{9}$$

where 3D Legendre moments, L_{pqr}, are computed over the same cube. If only Legendre moments of order smaller than or equal to Max are given, then the function $f(x,y,z)$ in Eq. (9) can be approximated as follows:

$$\hat{f}_{Max}(x,y,z) \approx \sum_{p=0}^{Max} \sum_{q=0}^{p} \sum_{r=0}^{q} L_{p-q,q-r,r} P_{p-q}(x) P_{q-r}(y) P_r(z). \tag{10}$$

The number of moments used in this form for image reconstruction is defined by:

$$N_{total} = \frac{(Max+1)(Max+2)(Max+3)}{6}. \tag{11}$$

2.2. Translation invariance of 3D Legendre moments

Central 3D Legendre moments are translation invariants. These moment invariants of order $(p + q + r)$ are defined as:

$$\varphi_{pqr} = \lambda_{pqr} \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} P_p(x - x_0) P_q(y - y_0) P_r(z - z_0) f(x,y,z) dxdydz. \tag{12}$$

The centroid of the 3D image/object, (x_0,y_0,z_0), is computed from the intensity function as follows:

$$x_0 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} x_i f(x_i,y_j,z_k) \Bigg/ \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} f(x_i,y_j,z_k), \tag{13.1}$$

$$y_0 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} y_j f(x_i,y_j,z_k) \Bigg/ \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} f(x_i,y_j,z_k), \tag{13.2}$$

$$z_0 = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} z_k f(x_i,y_j,z_k) \Bigg/ \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k}^{K} f(x_i,y_j,z_k). \tag{13.3}$$

To compute these central 3D moments, Ong et al. (2006) expressed the translated Legendre polynomials in terms of the original Legendre polynomials according to the relation:

$$P_p(x - x_0) = \sum_{k=0}^{p} \upsilon_{p(p-k)} P_{p-k}(x), \tag{14.1}$$

$$P_q(y - y_0) = \sum_{m=0}^{q} \tau_{q(q-m)} P_{q-m}(y), \tag{14.2}$$

$$P_r(z - z_0) = \sum_{\ell=0}^{r} \eta_{r(r-\ell)} P_{r-\ell}(z). \tag{14.3}$$

According to the conditions, $k - S1 = even$, $k - S2 = even$, the matrix $\upsilon_{p(p-n)}$ is defined by using the following equations:

$$\upsilon_{PP} = 1, \tag{15.1}$$

$$\upsilon_{p(p-k)} = \frac{1}{B_{(p-k)(p-k)}} \times \left[\sum_{S1=1}^{k} \binom{p-k+S1}{S1} (-x_0)^{S1} B_{p(p-k+S1)} - \sum_{S2=1}^{k-1} \upsilon_{p(p-S2)} B_{(p-S2)(p-k)} \right]. \tag{15.2}$$

The matrices $\tau_{q,(q-m)}$ and $\eta_{r,(r-\ell)}$ are defined by using similar equations. Based on Eqs. (15), it is clear that, the matrices $\upsilon_{p(p-n)}$, $\tau_{q,(q-m)}$