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Fully deformable 3D digital partition model with topological control

Guillaume Damiand [a,*], Alexandre Dupas [b], Jacques-Olivier Lachaud [c]

[a] Université de Lyon, CNRS, LIRIS, UMR5205, F-69622, France
[b] Université de Poitiers, CNRS, SIC-XLIM, UMR6172, F-86962, France
[c] Université de Savoie, CNRS, LAMA, UMR5127, F-73376, France

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A B S T R A C T

We propose a purely discrete deformable partition model for segmenting 3D images. Its main ability is to maintain the topology of the partition during the minimization process. To do so, our main contribution is a new definition of multi-label simple points (ML simple point) that is easily computable. An ML simple point can be relabeled without modifying the overall topology of the partition. The definition is based on intervoxel properties, and uses the notion of collapse on cubical complexes. This work is an extension of a former restricted definition (Dupas et al., 2009) that prohibits the move of intersections of boundary surfaces. A deformation process is carried out with a greedy energy minimization algorithm. A discrete area estimator is used to approach at best standard regularizers classically used in continuous energy minimizing methods. We illustrate the potential of our approach with the segmentation of 3D medical images with known expected topology.

1. Introduction

1.1. Context and contribution

Over the past 20 years, energy-minimizing techniques have shown a great potential for image segmentation. Originally, most of them were based on a variational formulation, i.e. a continuous optimization problem in a functional space. We may quote deformable models (Kass et al., 1988), Mumford–Shah approximation (Mumford and Shah, 1989), geometric or geodesic active contours and other levelset variants (Caselles et al., 1993, 1997; Malladi et al., 1995; Vese and Chan, 2002), among others. Their formulation combines in a single expression a term expressing the fit to data and a term describing shape priors (generally length or area penalization) and acting as a regularizer. The parameter balancing the two terms allows to tune the technique according to the amount of noise and perturbation in the data. In a sense, this parameter acts as a scale factor, providing a very natural multiscale analysis of images.

Energy-minimization for image segmentation can also be expressed in a discrete setting: structural split and merge (Dupas and Damiand, 2008), weighted graph with cut optimization (Boykov et al., 2001; Boykov and Kolmogorov, 2003), irregular and combinatorial pyramids (Guigues et al., 2003; Pruvot and Brun, 2007), Markov fields and stochastic processes (Geman and Geman, 1987), minimum description length (Leclerc, 1989; Zhu and Yuille, 1996). The discrete approaches present several advantages for finding the optimal solution. Greig et al. (1989) have given a polynomial algorithm for solving the two label segmentation problem. Approximate solutions for the multi-label partition are also available (Boykov et al., 2001; Guigues et al., 2006; Pruvot and Brun, 2007). However, the regularization/shape prior term of these discrete methods is reduced to the digital length or area of region boundaries, which is a very poor area estimator. Therefore, from the regularization point of view, it tends to flatten optimal configurations. As a consequence, optimal solutions may be geometrically somewhat different.

We propose a novel energy-minimizing model for segmenting 3D images into multiple regions. It aims at combining the advantages of the continuous and the discrete energy-minimizing techniques. This paper is an extension of the work of (Dupas et al., 2009). It shares with it the following features:

- **discrete model**: it is a purely digital formulation of energy minimization, which can be solved by combinatorial algorithms. In this exposition, we have for now use a simple greedy algorithm.
- **approximation of continuous regularization**: the area regularizer is approached in this digital setting by an accurate discrete geometric estimator.
- **contour-based and region-based energies**: both region structures and the geometry of their interfaces are encoded in the topological map structure. Any kind of energy may thus be evaluated efficiently: e.g. region-based like quadratic deviation

* Corresponding author. Tel.: +33 0 4 72 43 26 62; fax: +33 0 4 72 43 15 36.
E-mail addresses: guillaume.damiand@liris.cnrs.fr (G. Damiand), dupas@sic.univ-poitiers.fr (A. Dupas), jacques-olivier.lachaud@univ-savoie.fr (J.-O. Lachaud).

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(Mumford and Shah, 1989; Chan et al., 2001) or contour-based like strong gradients (Kass et al., 1988).

topological control: it is guaranteed that the topology of the whole partition remains unchanged during the evolution of the boundaries between regions.

Furthermore, compared with (Dupas et al., 2009), this paper describes a new method to guarantee that the topology of the whole partition is preserved during the deformation process, which allows a larger class of partition deformation at each step while guaranteeing that the partition topology remains unchanged.

1.2. Discussion

Our objective is to mimick as precisely as possible the behavior of continuous models while staying in a discrete setting. It is well known that continuous variational problems induce PDEs which are solved iteratively. They are dependent on their initialization and may get stuck in local minima, except in specific cases (Cohen and Kimmel, 1997; Chan et al., 2001; Ardon and Cohen, 2006). To our knowledge, none of them are able to find the optimal image partition if more than two regions are expected, although recent works using convex relaxation seem promising for 2D images (Chambolle et al., 2008; Pock et al., 2009). As said above, the discrete methods have interesting properties for extracting an optimal solution, but their regularization term is too primitive. As an exception, Boykov and Kolmogorov (2003) have proposed to enrich the neighborhood graph to get finer area estimators—in a way similar in spirit to chamfer distances—but their approach is for now limited to a 26-neighborhood, which remains a coarse approximation.

Our discrete model is related to the discrete deformable boundaries (Lachaud and Vialard, 2001), or to the discrete snake (de Vieilleville and Lachaud, 2009). Instead of enriching the adjacency graphs, we keep the standard image graph but we compute the regularization term in a potentially larger neighborhood with discrete geometric estimators. This term is an estimation of the area of each surfel. The discrete geometric estimator extracts maximal digital straight segments along two directions to estimate the surfel normal, the surfel area is then a byproduct (Lachaud and Vialard, 2003). Such estimators are known to have good convergence behavior as the resolution gets finer and finer (Lachaud et al., 2007). In a sense, the discrete energy tends toward the continuous energy as the resolution gets finer. This is proved for the 2D formulation in (Lachaud, 2006). In the present paper, we use only greedy combinatorial optimization schemes, which entails that our model may also be stuck in local minima, but the proposed framework let us free to test more elaborate combinatorial optimization algorithm.

This model encodes the 3D evolving digital partition with a combinatorial map, which offers a simple and optimal access to the partition topology. Regions are then naturally delineated and region energies are easily computed. This partition model also encodes the digital geometry between regions with an intervoxel matrix. Frontiers can be tracked in a straightforward way to compute contour-based energies. As a consequence, we obtain a versatile segmentation tool. According to the image characteristics or the application, it is well known that contour or region based approaches are more or less adapted. For instance, region-based energies are generally more convex and thus easier to minimize (Chan et al., 2001; Vese and Chan, 2002). Our partition model allows to mix energies defined on regions and energies defined on boundaries. Very few explicit or implicit variational or deformable models can do that in 3D, except perhaps the work of (Pons et al., 2007), but their approach may not model energies depending on the inclusion between regions.

Finally, we address the problem of controlling the topology of the partition while it is evolving toward a minimal position. This is critical in several specific image applications where the topology of anatomic components is a prior information, like atlas matching. This is even truer in 3D images, where anatomic components are intertwined in a deterministic way. In 2D, when there are only two labels (foreground and background), simple points are a classical technique for doing it (Bertrand, 1994). Similar tools are used in level set techniques to control topology changes (Han et al., 2003; Ségonne, 2008). For the more complex case of a multi-label partition, a few authors have proposed an equivalent to simple points in a discrete setting (Ségonne et al., 2005; Bazin et al., 2007). However, they are computationally too costly to be used to drive the evolution of a digital partition.

This paper is an extension of the work (Dupas et al., 2009), where a first notion of simple point in a partition was proposed. This first definition was enough to simulate movements of boundaries between two regions, but it forbade movements of boundaries between three or more regions (1-dimensional boundaries). We propose here a more general definition of simple points in multi-label partitions, which we call *ML-simple points* (ML for multi-label). This new definition gives more freedom to the evolving partition. Updating ML-simple points induces movements of surface, edges, and points between regions, while preserving at all steps the initial partition topology. Moreover, ML-simpleness is computable in constant time, thanks to our intervoxel encoding.

The paper is organized as follows. Section 2 recalls standard notions of digital geometry used later on. Section 3 presents the definition of ML-simpleness and proves that it implies simpleness. The ML-simpleness test derives from the definition. Section 4 describes a first digital deformable partition model that uses ML-simple points to ensure the preservation of the topology and Section 5 shows some experiments.

2. Preliminary notions

The first subsection recalls standard digital topology notions based on voxels. The second subsection gives further definitions for intervoxel topology. The third subsection presents the definitions related to cubical cell complexes and the last subsection gives our first restricted version of ML-simpleness.

2.1. Images and voxels notions

A *voxel* is an element of the discrete space $\mathbb{Z}^3$. A 3D image is a finite set of voxels I (the image domain), and a mapping between these voxels and a set of colors or a set of gray levels (the image values). Each voxel v is associated with a label $l(v)$, a value in a given finite set L. These labels can be obtained from the image by a segmentation algorithm.

We use the classical notion of α-*adjacency*, with $\alpha \in \{6, 18, 26\}$. The set of voxels α-adjacent to v is noted $N^*_\alpha(v)$, and thus we define $N_\alpha(v) = N^*_\alpha(v) \cup \{v\}$. An α-*path* between two voxels v_1 and v_2 is a sequence of voxels between v_1 and v_2 such that each pair of consecutive voxels is α-adjacent. A set of voxels S is α-*connected* iff there is an α-path between any pair of voxels of S, having all its voxels in S.

We consider the relation induced by being 6-connected and having the same label. This is an equivalence relation over the image domain, and the equivalence classes are the *regions* of the image. We consider an infinite region R_0 that "surrounds" the image (*i.e.* $R_0 = \mathbb{Z}^3 \setminus I$). Note that there is only one infinite region, which is not necessarily 6-connected if the image has some holes. The complement set of a region X in I is denoted by $\overline{X}$. We extend the notion of adjacency to regions: two regions R_1 and R_2 are