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Digital flow for shape decomposition: Application to 3-D microtomographic images of snow ☆

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ABSTRACT

We propose a fast shape decomposition method for granular microstructures using a 3-D approach based on medial axis. We define a two-step algorithm: the first step relies on a notion of digital flow to obtain a preliminary over-decomposition from medial balls. During a second step, we use geometric criteria to obtain a relevant and precise volumetric decomposition. We apply our algorithm to 3-D objects of materials and, more precisely, to microtomographic images of snow microstructures.

1. Introduction

Shape decomposition is one of the fundamental techniques in computer graphics and is widely used in shape processing. The goal of decomposition, sometimes called *segmentation*, is to simplify and/or change the representation of an object in order to make it more meaningful and easy to analyze [33]. The principal contribution of this paper focuses on a fast and efficient shape decomposition method which is based on the digital flow. The concept of flow was introduced in [12]. With the proposition of a fast computation of critical points in digital domain, we obtain a framework of method which is optimal in time. Moreover, we provide two distinct geometrical criteria to control the quality of the decomposition.

In this paper, we first propose a digital version of the flow notion from computational geometry to yield a fast initial decomposition of 3-D granular materials into regions (Section 4). This approach provides a structure on the initial regions which allows us to define a simple filtering algorithm to correct over-decomposition effects (Section 5). We validate the quality of the decomposition on both synthetic data and images of granular snow samples (Section 6).

2. Related works

The main context of this paper is the analysis of granular materials from 3-D computed tomographic images. More precisely, we focus on a specific granular material, i.e. *deposited snow on the ground* (see Fig. 9), which is observed at the scale of its *microstructure* (1 voxel $\sim$ 5–20 μm). In this context, micro-scale modelling requires a precise 3-D description of snow microstructures in terms of individual grains and bond's characteristics [6]. Practically, there are various shape types of snow present in the snowpack, like *Precipitation Particles* (PP), *Rounded Grains* (RG), *Melt Forms* (MF) and so on [15]. Each class implies different geometry of grains from nearly spherical objects to facetted ones. So the challenge is to decompose the 3-D images of these different snow types into grains, which are usually sintered together and form complex shapes. Another specific aspect of our context is that physical analysis of snow microstructures leads to further requirements on the grain-to-grain interfaces: the interface between two grains should be flat or with minimal curvature values. We do not use such an information directly in our segmentation approach but we rely on it in our experimental evaluation.

From image processing, several approaches for shape decomposition problems consider mathematical morphology tools such as watershed transform [13] or region growing operators [32]. In our context where the input object is a binary volume, the main idea of these approaches is to start from a set of markers defined by local maxima in the distance transform of the input shape (see [34,22] for a survey). Then, a propagation process is used to enlarge catchment basins of each local minimum to define the

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overall decomposition into non-overlapping regions. Despite several improvements [14], the main drawback is that such approaches have difficulties to capture the complex shape geometry of snow grains and bonds.

Surface based techniques can also be considered. The main idea is to perform a first decomposition on the 3-D object boundary and to propagate such decomposition to the object's interior to finally obtain the volumetric decomposition. If we suppose that grains are smooth with rounded shapes, differential estimators (mean and Gaussian curvatures) can be used to decompose the surface into components with almost constant curvature values [37]. In a previous work [36], we have developed such decomposition tools based on surface curvature information. This method identifies groove regions on the surface of object to locate the possible separating boundaries in volume. However, all these techniques are highly sensitive to the initial surface decomposition into groove regions from curvature map. Furthermore, they require stable and robust to noise differential curvature estimators, which could be challenging.

Another approach consists in decomposing the initial shape using volumetric information based on the distance map [35] or the medial axis representation of a shape [12]. For the first mentioned approach, the idea is close to the watershed approach: we start from local maxima of the distance map and we perform a propagation process to construct the regions. A last step is required to overcome the over-decomposition induced by the first step and uses a heuristic based merging process between adjacent regions. Similarly to watershed, the method is highly sensitive to the initial local maxima computation and the region interface quality is poor. From computational geometry, Dey et al. [12] proposed an interesting mathematical tool which constructs a continuous flow from the medial axis representation of a shape. In this approach, the object is represented by point sets on its boundary and the medial axis is defined as a subset of the Voronoi diagram of the input point set [3]. Another method which is based on curve skeletons was proposed by Reniers and Telea [27,28]. The curve-skeleton junctions which signal the interpenetration of parts are detected based on the junction rule using a function based geodesic metric to quantify the relevance of a given curve-skeleton branch. These approaches provide very good results on 3-D models and CAD shapes. However, when applying them to large microtomographic images of snow microstructures (high resolution objects, high topology genus, noisy curve-skeleton with small shortest loops associated to surface), these approaches become time consuming and may lead to inconsistent decomposition.

We propose here a purely volumetric approach which does not require to back-project volumetric information (curve-skeleton or medial structures) to the object surface to compute geometrical information. Our proposal is thus based on simple digital volumetric data structures (digital power map and digital flow), which can be obtained by very fast algorithms.

3. Preliminaries

In this section, we outline the notion of *Flow* induced by a shape [12]. The original *Flow* definition is described here in a more general setting by considering general shapes which are embedded in d-dimensional Euclidean space $\mathbb{R}^d$.

3.1. Flow in continuous space

In the following, $\mathcal{X}$ denotes a compact subset of d-dimensional Euclidean space $\mathbb{R}^d$, $\partial\mathcal{X}$ denotes its boundary. The definitions can be found in [12]. Given $\mathcal{X} \subset \mathbb{R}^d$, the *distance transform* $h : \mathbb{R}^d \rightarrow \mathbb{R}$ is defined at each point $x \in \mathbb{R}^d$ such that

$$h(x) = inf_{y \in \partial\mathcal{X}} \|y - x\|^2 \qquad (1)$$

Definition 1 (*Anchor set*). For all $x \in \mathbb{R}^d$, the anchor set $A(x)$ of x is given by

$$A(x) = argmin_{y \in \partial\mathcal{X}} \|y - x\|^2 \qquad (2)$$

In other words, $A(x)$ is the set of the closest points to x in $\partial\mathcal{X}$. Let $conv(A(x))$ be the convex hull of $A(x)$. In Fig. 1, we illustrate, in dimension 2, several configurations where $conv(A(x))$ is a triangle or an edge.

Definition 2 (*Critical and Regular points*). A point $x \in \mathbb{R}^d$ is a *critical point* if $x \in conv(A(x))$. Otherwise, x is regular.

The flow is defined by using the direction of steepest ascent. First, we set $d(x)$ as driver of x, where $d(x) = argmin_{y \in conv(A(x))} \|y - x\|^2 \; \forall \; x \in \mathbb{R}^d$. We then define a vector $v : \mathbb{R}^d \rightarrow \mathbb{R}^d$, $v(x) = \frac{x - d(x)}{\|x - d(x)\|}$ if $x \neq d(x)$ and 0 otherwise.

Definition 3 (*Induced Flow*). The flow is a function $\phi : [0, \infty) \times \mathbb{R}^d \rightarrow \mathbb{R}^d$, the right derivative of which satisfies, at each point $x \in \mathbb{R}^d$

$$\lim_{t \downarrow t_0} \frac{\phi(t, x) - \phi(t_0, x)}{t - t_0} = v(\phi(t_0, x)) \qquad (3)$$

Definition 4 (*Stable manifold*). The stable manifold $S(x)$ of a critical point x is the set of all the points which flow into x.

$$S(x) = \{y \in \mathbb{R}^d : \lim_{t \rightarrow \infty} \phi(t, y) = x\} \qquad (4)$$

The stable manifolds of all critical points induce a decomposition of the object into disjoint regions (the word *stable* thus refers to locii where the flow gradient is null). It means, $\mathbb{R}^d = \bigcup_x S(x)$ for all critical points x. Furthermore, the decomposition is valid since for any two critical points x and y ($x \neq y$), we have $S(x) \cap S(y) = \emptyset$.

3.2. Medial axis and digital medial axis

The Medial axis of a shape is a classic method for shape analysis. It was first proposed by Blum [4] in the continuous plane and can be defined as the set of balls contained in $\mathcal{X}$ touching at least twice $\partial\mathcal{X}$. Following previous definitions, a ball with center $x \in \mathcal{X}$ and radius r belongs to the medial axis if and only if $|A(x)| \geqslant 2$ and $\|y - x\| = r$ for any point $y \in A(x)$.

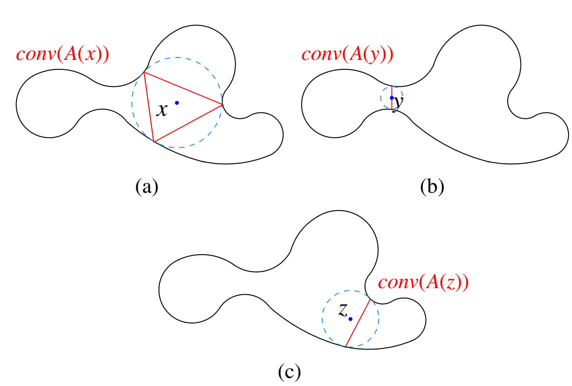

Fig. 1. Several configurations to illustrate the definition of critical points: In (a), x is such that $x \in conv(A(x))$ (triangle in red) and is thus a critical point. In (b), y lies in the segment $conv(A(y))$, y is a critical point too. In (c), $z \not\subseteq conv(A(z))$, so z is a regular point. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)