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Fractal descriptors based on the probability dimension: A texture analysis and classification approach ☆

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ABSTRACT

In this work, we propose a novel technique for obtaining descriptors of gray-level texture images. The descriptors are provided by applying a multiscale transform to the fractal dimension of the image estimated through the probability (Voss) method. The effectiveness of the descriptors is verified in a classification task using benchmark over texture datasets. The results obtained demonstrate the efficiency of the proposed method as a tool for the description and discrimination of texture images.

1. Introduction

Fractals have played an important role in many areas with applications related to computer vision and pattern recognition [1–6], owing to their flexibility in representing structures usually found in nature. In such objects, we observe different levels of detail at different scales, which are described in a straightforward manner by fractals, rather than through classical Euclidean geometry.

Most fractal-based techniques are based on the concept of fractal dimension. Although this concept was originally defined only for mathematical fractal objects, it contains some properties that make it a very interesting descriptor for any object in the real world. Indeed, fractal dimension measures how the complexity (level of detail) of an object varies with scale, an effective and flexible means of quantifying how much space an object occupies, as well as important physical and visual properties of the object, such as luminance and roughness.

Fractal techniques include the use of Multifractals [7–9], Multiscale Fractal Dimension [10,11] and fractal descriptors [12–16]. Here we are focused on the last approach, which has demonstrated the best results in texture classification [17]. The main idea of fractal descriptors theory is to provide descriptors of an object represented in a digital image from the relation among fractal dimensions taken at different observation scales, thus these values provide a valuable information on the complexity of the object, in the sense that they capture the degree of detail at each scale. In this way, fractal descriptors are capable of quantifying important physical characteristics of the structure, as the fractal dimension, but presenting a richer information than can be provided by a single number (fractal dimension).

Although fractal descriptors have demonstrated to be a promising technique, we observe that they are defined mostly on well-known methods to estimate the fractal dimension. Here, we propose fractal descriptors based on a less known definition of fractal dimension: the probability dimension. This is a statistical approach, which measures the distribution of pixel intensities along the image. In this way, such descriptors can express how the statistical arrangement of pixels in the image changes with the scale and how much such correlation approximates a fractal behavior. In this sense, our descriptor also measure the self-similarity and complexity of the image but upon a statistical viewpoint. This is a rich and not explored perspective, which is studied in depth in this work.

We use the whole power-law curve of the dimension and apply a time-scale transform to emphasize the multiscale aspect of the features. Finally, we test the proposed method over three well-known datasets, that is, Brodatz, Outex and UIUC, comparing the results with another fractal descriptor approach showed in [13] and other conventional texture analysis methods. The results demonstrate that probability descriptors achieve a more precise classification than other classical techniques.

2. Fractal theory

In recent years, fractal geometry concepts have been applied to the solution of a wide range of problems [1–6], mainly because

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conventional Euclidean geometry has severe limitations in providing accurate measures of real-world objects.

2.1. Fractal dimension

The first definition of fractal dimension provided in [18], is the Hausdorff dimension. In this definition, a fractal object is a set of points immersed in a topological space. Thus one can use results from measure theory to define a measure over this object. This is the Hausdorff measure expressed by

$$H^s_\delta(X) = \inf \sum\nolimits_{i=1}^{\infty} |U_i|^s : U_i \text{ is a } \delta\text{-cover of } X, \tag{1}$$

where $|X|$ denotes the diameter of X, that is, the maximum possible distance among any elements of X:

$$|X| = \sup\{|x-y| : x, y \in X\}. \tag{2}$$

Here, a countable collection of sets U_i, with $|U_i| \leqslant \delta$, is a δ-cover of X if $X \subset \cup_{i=1}^{\infty} U_i$.

Notice that H also depends on a parameter δ, which expresses the scale at which the measure is taken. We can eliminate such dependence by applying a limit over δ, defining in this way the s-dimensional Hausdorff measure:

$$H^s(X) = im_{\delta \to 0} H^s_\delta(X). \tag{3}$$

The plot of $H^s(X)$ as a function of s shows a similar behavior in any fractal object analyzed. The value of H is ∞ for any $s < D$ and it is 0 for any $s > D$, where D always is a non-negative real value. D is the Hausdorff fractal dimension of X. More formally,

$$D(X) = \{s\} | \inf\{s : H^s(X) = 0\} = \sup\{H^s(X) = \infty\}. \tag{4}$$

In most practical situations, the Hausdorff dimension is difficult or even impossible to calculate. Thus assuming that any fractal object is intrinsically self-similar, the literature shows a simplified version, also known as the similarity dimension or capacity dimension:

$$D = -\frac{\log(N)}{\log(r)}, \tag{5}$$

where N is the number of rules with linear length r used to cover the object.

In practice, the above expression may be generalized by considering N to be any kind of self-similarity measure and r to be any scale parameter. This generalization has given rise many methods for estimating fractal dimension, with widespread applications to the analysis of objects that are not real fractals (mathematically defined) but that present some degree of self-similarity in specific intervals. An example of such a method is the probability dimension, used in this work and described in the following section.

2.2. Probability dimension

The probability dimension, also known as the information dimension, is derived from the information function. This function is defined for any situation in which we have an object occupying a physical space. We can divide this space into a grid of squares with side-length δ and compute the probability p_m of m points of the object pertaining to some square of the grid. The probability function is given by

$$N_P(\delta) = \sum_{m=1}^{N} \frac{1}{m} p_m(\delta), \tag{6}$$

where N is the maximum possible number of points of the object inside a unique square. Here we use a generalization of the above expression defined in the multifractal theory [19]:

$$N_P(\delta) = \sum_{m=1}^{N} m^\alpha p_m(\delta), \tag{7}$$

where α is any real number.

The dimension itself is given as

$$D = -\lim_{\delta \to 0} \frac{\ln N_P}{\ln \delta}. \tag{8}$$

When this dimension is estimated over a gray-level digital image $I : [M, N] \to \mathfrak{R}$, a common approach is to map it onto a three-dimensional surface S as

$$S = \{i, j, I(i,j) | (i,j) \in [1:M] \times [1:N]\}. \tag{9}$$

In this case, we construct a three-dimensional grid of 3D cubes also with side-length δ. The probability p_m is therefore given by the number of grid cubes containing m points on the surface divided by the maximum number of points inside a grid cube (see Fig. 1).

3. Fractal descriptors

Fractal descriptors are values extracted from the log–log relationship common to most methods of estimating fractal dimension. Actually, any fractal dimension method derived from the concept of the Hausdorff dimension obeys a power-law relation, which may be expressed as

$$D = -\frac{\log(\mathfrak{M})}{\log(\epsilon)}, \tag{10}$$

where $\mathfrak{M}$ is a measure depending on the fractal dimension method and ϵ is the scale at which this measure is taken.

Therefore Fractal descriptors are provided from the function u:

$$u : \log(\epsilon) \to \log(\mathfrak{M}). \tag{11}$$

We call the independent variable t to simplify the notation. Thus $t = \log \epsilon$ and our fractal descriptor function is denoted $u(t)$. For the probability dimension used in this work, we have

$$u(t) = -\frac{\log(N_V(\delta))}{\log(\delta)}. \tag{12}$$

The values of $u(t)$ may be directly used as descriptors of the analyzed image or may be post-processed by some kind of operation aimed at emphasizing some specifical aspects of that function. Here, we apply a multiscale transform to $u(t)$ and obtain a bidimensional function $U(b, a)$, in which the variable b is related to t and a is related to the scale at which the function is observed. A common means of obtaining U is through a wavelet transform:

$$U(b,a) = \frac{1}{a} \int_{\mathfrak{R}} \psi\left(\frac{t-b}{a}\right) u(t) dt, \tag{13}$$

where ψ is a wavelet basis function and a is the scale parameter [20]. Fig. 2 shows an example where two textures with the same dimension, but visually distinct, provide different descriptors.

4. Proposed method

This work proposes to obtain fractal descriptors from textures by using the probability fractal dimension. At first, the values on the curve $u(t) : \log(N_P(\delta))$ in Eq. (7) are computed for each image. Therefore we apply a multiscale transform to u.

The multiscale process employs a wavelet transform of $u(t)$, as described in the previous section:

$$U(b,a) = \frac{1}{a} \int_{\mathfrak{R}} \psi\left(\frac{t-b}{a}\right) u(t) dt. \tag{14}$$