Pattern Recognition Letters 59 (2015) 1–10

Contents lists available at ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

Self-affine snake for medical image segmentation☆

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ARTICLE INFO

Article history:
Received 3 August 2013
Available online 17 March 2015

Keywords:
Parametric active contours
Wavelet transform
Self-affine mapping system
Medical image segmentation

ABSTRACT

In this paper, a new parametric active contour called self-affine snake is proposed for medical image segmentation. It integrates the wavelet transform, parametric active contour (or snake), and self-affine mapping system to keep their strengths and avoid the weak points. In more detail, it inherits wide capture range from wavelet transform and topological consistency from snake. Furthermore, it takes advantage of self-affine mapping system in several aspects including (i) convergence to weak boundaries, especially, next to strong edges, (ii) reconstruction of boundary openings, and (iii) progress into boundary concavities. The experimental results were performed using a number of synthetic and medical images given in five sets of experiments. Self-affine snake provided comparable/superior results in terms of both solution quality and CPU time compared to a number of frequently-used active contours including balloon, gradient vector flow (GVF), generalized GVF, and active contour without edges. However, its most important properties were the significant robustness against noise and reconstruction of boundary openings. Because of the valuable advantages, the proposed algorithm is an appropriate approach, particularly, for segmentation of medical images which usually suffers from noise corruption and edge uncertainty.

1. Introduction

Segmented medical images are used routinely in a multitude of different applications such as study of anatomical structure [50], diagnosis [2], and localization of pathology [29]. However, medical image segmentation remains a difficult task due to both tremendous variability of object shapes and image corruption by noise and artifacts. Active contours (snakes) have been introduced as a solution [51].

1.1. Explicit and implicit active contours

Generally speaking, active contours are deformable models which can deform/evolve in the image domain in order to minimize internal and external energies. The internal energy makes the curve smooth, while the external energy moves it toward the interesting features in the image domain. Active contours can be roughly divided into two categories: parametric (PAC) and geometric/geodesic active contours (GAC). Each PAC is explicitly represented as a parameterized curve in a Lagrangian formulation [20] while every GAC is implicitly represented as a level set of a two-dimensional function [5,30]. GACs can evolve based on the surface evolution theory and geometric flows, mainly, in light of the Euler formulation.

Compared to GACs, PACs are computationally simpler and more efficient [17]. Also, user interaction and *a priori* shape-constraints establishment are more straightforward due to explicit representation of the curve [22]. Furthermore, they usually perform better in boundary gaps [14]. However, in contrast, GACs have the ability of handling topological changes which makes them preferable for segmentation of complex shapes [19]. Also, although tuning of their parameters is usually easier, the stopping criteria of level set methods may be partially established based on the number of iterations [35].

In another aspect, deformable models can be divided into two different categories: edge-based and region-based active contours (EBAC and RBAC, respectively). EBACs use local edge-information to attract the curve toward the object boundaries [36,53,54] or stop its evolution [5,10]. They are partially sensitive to noise and initialization [14]. RBACs have been proposed to tackle these problems [6,37]. Their main contribution is identifying each region of interest through statistical descriptors and homogeneity requirements in order to evolve the curve. However, these models may not be able to localize object boundaries as well as EBACs. To address this problem, some researchers tried to integrate the edge and region-based models [4,21,37]. Despite promising results, the conventional RBACs may fail to segment images with texture, inhomogeneous intensity, noise corruption, or heterogeneous objects. Recently, patch-based variational formulations have been proposed as a solution to that problem [11,24,26,32,46]. Instead, a large number of researchers incorporate *a priori* shape information or statistical features into the energy functional to effectively tackle the above challenge [1,12,13,39,45].

☆ This paper has been recommended for acceptance by A. Heyden.

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http://dx.doi.org/10.1016/j.patrec.2015.03.006
0167-8655/

1.2. Relationship between PACs and GACs

A traditional PAC is a parametric curve ($\boldsymbol{x}(s) = [x(s), y(s)], s \in [0, 1]$) which moves in the spatial domain to minimize the following energy functional [20]:

$$e = \frac{1}{2}\int_0^1 \alpha\left|\frac{\partial \mathbf{x}}{\partial s}\right|^2 + \beta\left|\frac{\partial^2 \mathbf{x}}{\partial s^2}\right|^2 ds + \int_0^1 p\left(\boldsymbol{x}(s)\right) ds \tag{1}$$

where α and β are two scalar constants, s is the arc-length, and $|\cdot|$ computes the norm of a vector. The potential function $p(\cdot)$ takes on smaller values at features of interest in the image (*e.g.* object boundaries). In the above equation, the first and second integrals indicate the internal and external energies, respectively. The former controls the snake tension (α) and rigidity (β) while the latter is derived from the image data. For example, the frequently-used Gaussian external energy can be given by using the following potential function:

$$p_{\mathrm{GSN}}(x, y) = -\left|\nabla\left[G_\sigma(x, y) * I(x, y)\right]\right|^2 \tag{2}$$

where $G_\sigma(x, y)$ is a two-dimensional Gaussian kernel with standard deviation of σ; and ∇ is the gradient operator. An optimal snake which minimizes e must satisfy the Euler–Lagrange equation. Thus, we can write:

$$\frac{\partial \boldsymbol{x}}{\partial t} = \boldsymbol{f}_{\mathrm{int}} + \boldsymbol{f}_{\mathrm{ext}}^{(\mathrm{p})} \tag{3}$$

where $\boldsymbol{f}_{\mathrm{int}} = \alpha\frac{\partial^2 \boldsymbol{x}}{\partial s^2} - \beta\frac{\partial^4 \boldsymbol{x}}{\partial s^4}$ and $\boldsymbol{f}_{\mathrm{ext}}^{(\mathrm{p})} = -\nabla p$ are the internal- and external-potential forces, respectively. Furthermore, in the above equation, by using a dynamic force formulation, the potential force $\boldsymbol{f}_{\mathrm{ext}}^{(\mathrm{p})}$ can be replaced by the general force $\boldsymbol{f}_{\mathrm{ext}}^{(\mathrm{g})}$ which is usually a combination of potential and non-potential forces [54].

Furthermore, Xu et al. [51] stated that every external force can be equivalently employed (with $\beta = 0$) to evolve a GAC as follows:

$$\gamma\frac{\partial \varphi}{\partial t} = (\alpha\kappa + \omega_p)|\nabla\varphi| - \boldsymbol{f}_{\mathrm{ext}}^{(\mathrm{g})} \cdot \nabla\varphi \tag{4}$$

where φ and κ are the level-set function and its curvature, respectively; ω_p indicates the constant deformation speed; and γ is a scalar constant. Hereafter, we will focus on edge-based PACs considering that the proposed method essentially belongs to that category of snakes.

1.3. Edge-based parametric active contours

Edge-based PACs suffer a number of major drawbacks such as dependency on the initial contour, difficult reconstruction of edge openings, hard progressing into boundary concavities, marching over weak edges (especially, next to strong edges), and invalid results for inhomogeneous and noisy regions [9,53]. Most methods introduced to address these drawbacks solve one problem while making new inconveniences. For example, multiresolution approaches increase the capture range while moving the contour across different resolutions remains a challenging task [8,10]. Balloon addresses both the capture range and boundary concavity problems although weak edges may be overwhelmed by using a too strong pressure force [9]. Furthermore, it requires careful initialization because the pressure force is unidirectional. Park et al. [38] improved the snake performance by integrating both the gradient direction and strength in the energy functional. Nascimento and Marques [34] proposed adaptive snakes using expectation maximization algorithm to avoid undesirable edge points. Zhu et al. [55] proposed effective external forces by incorporating geometric information of an edge map into a virtual electric field model. Another example is gradient vector flow (GVF) which effectively tackles the capturing range and boundary concavity problems by minimizing an energy-functional [54]. Researchers proposed a number of variations on GVF to improve its performance in various aspects ([7,18,27,28,36,40,49,52]).

1.4. Proposed active contour

Considering the aforementioned difficulties of active contours, some researchers proposed several contour extraction algorithms which may inherently differ from snakes [48]. For example, Ida and Sambonsugi [15] presented a highly accurate method to approach and fit a roughly drawn line to the object contour by using the self-affine mapping system (SAMSYS). However, sometimes, it abnormally deforms the curve due to the fractal behavior.

The authors addressed this drawback by taking care of the curve topology [44]. Besides, they employed contractive self-affine maps (CSAMs) to propose self-affine snake (SAS) for medical images segmentation [41,43]. This paper builds on our previous researches and its main contributions are as follows:

- Establishing a comprehensive theoretical basis behind the design of our algorithm and presenting the related technical details.
- Improving the cost function of CSAMs to avoid uncertainties caused by multiple equivalent optimal translation vectors.
- Enhancing the computational burden of the proposed active contour by using dynamically efficient implementation.
- Assessing the outstanding properties of the proposed active contour such as providing large capture range, handling boundary openings and concavities, preventing weak-edge leakage, and robustness against noise.
- Evaluating the performance of the proposed method compared to a number of counterpart active contours such as balloon, GVF, generalized GVF (GGVF) [53], and active contour without edges (ACWE) [6] in terms of both solution quality and CPU time.

1.5. Paper outline

The remainder of the paper is organized as follows. In Section 2, the self-affine mapping system is briefly introduced. In Section 3, we present the mathematical principles of self-affine snake including details of the main idea, improvements on the conventional cost function of CSAMs and dynamically efficient implementation. Section 4 is devoted to experimental results and finally, conclusions are drawn in Section 5.

The notations used in this paper are fairly standard. Matrices are shown by upper case letters while boldface symbols are used for vectors in lower case letters.

2. Self-affine mapping system

SAMSYS has been frequently used for fractal image coding [23] and producing fractal figures [3]. However, most applications of SAMSYS in the image processing domain including image segmentation, edge detection, and contour extraction were initiated by Ida and Sambonsugi [15,16]. All of these promising advances are usually based on the attracting/repelling behaviors of contractive/expanding self-affine maps which were experimentally demonstrated in [16] and analytically proved in [42]. However, this paper is focused on only CSAMs which are employed to provide self-affine forces.

2.1. Contractive self-affine maps

Consider an image having the domain $\Omega_i \subset R^2$ with the intensity $I(\boldsymbol{x}) \in [0, 1]$ for all $\boldsymbol{x} = (x, y) \in \Omega_i$. The ith CSAM (m_i:$\mathrm{M}_i \to \mathrm{W}_i$) is defined as follows:

$$\boldsymbol{x}_\omega = m_i(\boldsymbol{x}) = r_i(\boldsymbol{x} - \bar{\boldsymbol{x}}_i) + \boldsymbol{\tau}_i + \bar{\boldsymbol{x}}_i, \quad r_i < 1 \tag{5}$$

where $\bar{\boldsymbol{x}}_i$ is the center point of M_i and $\boldsymbol{\tau}_i = (s_i, t_i)$ is the corresponding translation vector which moves the center point of M_i to the center point of W_i. The above equation translates the domain-patch M_i by $\boldsymbol{\tau}_i$ and contracts it by the scaling coefficient r_i to make the range-patch W_i (both $\subset\Omega_i$). To reduce computational burden, all scaling