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Shape classification using complex network and Multi-scale Fractal Dimension

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ABSTRACT

Shape provides one of the most relevant information about an object. This makes shape one of the most important visual attributes used to characterize objects. This paper introduces a novel approach for shape characterization, which combines modeling shape into a complex network and the analysis of its complexity in a dynamic evolution context. Descriptors computed through this approach show to be efficient in shape characterization, incorporating many characteristics, such as scale and rotation invariant. Experiments using two different shape databases (an artificial shapes database and a leaf shape database) are presented in order to evaluate the method, and its results are compared to traditional shape analysis methods found in literature.

1. Introduction

Shape is a feature of great importance in human communication. It is one of the most important visual attributes of an object and the first used to perform object identification and classification (Loncaric, 1998; Torres et al., 2003).

Shape analysis is a classical problem in the pattern recognition area. Over the years, several methods for shape description and recognition have been suggested. Basically, these methods can be classified into three approaches: region-based, boundary-based and skeleton-based approaches. The region-based methods consider all image object to compute a set of descriptors which characterize that shape (Zhenjiang, 2000; Khotanzad and Hong, 1990; Hu, 1962). These methods can be applied to generic shapes, but they fail to distinguish among objects that are too similar. The boundary-based methods are specified and more efficient for shapes described by their contours (Mokhtarian and Bober, 2003). Methods such as Fourier descriptors (Mehtre et al., 1997; Wallace and Wintz, 1980; Osowski and Nghia, 2002), Curvature Scale Space (CSS) (Mokhtarian and Bober, 2003, 1993), wavelet descriptors (Chuang and Kuo, 1996), Multi-scale Fractal Dimension (Torres et al., 2003; Plotze et al., 2005; Bruno et al., 2008), Inner Distance (Ling and Jacobs, 2007) and Learning Graph Transduction (Yang et al., 2008) are included in this category. These techniques consider the shape as a set of ordered coordinates, and so, the lack of points, missing parts or even occlusion of a shape region could affect the results. Skeleton-based methods, such as Bai et al. (2008), Bai and Latecki (2008), consider only the media axis of the shape during its analysis. For this reason, these methods usually present better results than contour or others shape methods in the presence of occlusion and articulation of parts. Besides, the information about local thickness of the shape is lost during the skeleton computing. Such information is vital to distinguish solid shapes and linear shapes.

Multi-scale Fractal Dimension is a complexity analysis method which allows shape identification by analysing its irregularity pattern. It consists in estimating a curve that represents the changes in shape complexity as we change the visualization scale. Shape complexity is straight related to the irregularity pattern presented by the shape and, respectively, the amount of the space the shape occupies (Chaudhuri and Sarkar, 1995; Lange et al., 1996; Plotze et al., 2005; Emerson et al., 1999; Gonzalez and Woods, 2002; Tricot, 1995).

Presented paper proposes a novel approach to the shape boundary analysis problem using the Complex Network Theory (Albert and Barabási, 2002; Boccaletti et al., 2006; Costa et al., 2007; Dorogovtsev and Mendes, 2003) and Multi-scale Fractal Dimension (Torres et al., 2003; Plotze et al., 2005). Points in the shape contour are considered as a set of vertices and modeled as a network. Then, topological features, derived from the dynamics of the network growth, are correlated to physical aspects of the shape. Multi-scale Fractal Dimension curve is computed from degree measurements of the network. Result is a curve representing network complexity that may be used to identify and distinguish shape objects.

This paper is organized as follows: Section 2 presents an overview of the Complex Network Theory and some network measurements found in the literature. This section also details how to

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model a shape as a complex network. Section 3 shows how Fractal Dimension can be estimated from a complex network while Section 4 describes how the Multi-Scale Fractal Dimension is computed and how to compose a shape signature based in its descriptors. In Section 5, the performance of the proposed descriptors is evaluated using linear discriminant analysis (LDA) (Everitt and Dunn, 2001) in an experiment based on image classification. For this, two image databases were considered: (i) generic artificial shapes and (ii) plant leaves shapes. Proposed descriptors are also compared with traditional shape analysis methods found in Literature. Conclusions are finally presented in Section 6.

2. Complex network

Nowadays, complex networks have become a topic of great interest in many fields of science (Albert and Barabási, 2002; Newman, 2003; Dorogovtsev and Mendes, 2003). This interest is due to the capacity of complex networks represent many real-world systems, natural structures and computer vision topics. However, this is still an unexplored field with few references in the literature (Costa, 2004).

Studies conducted by Flory (1941), Rapoport (1951), Rapoport (1953), Rapoport (1957), Erdös and Rényi (1959), Erdös and Rényi (1960), Erdös et al. (1961) settle the basis of this research area, which can be understood as an intersection between graph theory and statistical mechanisms. This grants a truly multidisciplinary nature to this area (Costa et al., 2007). Recent motivation in complex network research is due to the investigations about Small-World Networks performed by Watts and Strogatz (1998) and Barabási and Albert characterizing Scale-Free models (Barabási and Albert, 1999).

The main reason for complex network popularity lays in its flexibility and generality to represent any given structure, natural or discrete (such as lists, trees, networks and images (Costa, 2004)), including those undergoing dynamic changes of topology (Costa et al., 2007).

Many papers use complex networks to represent real structures. These studies include investigations of the problem representation as a complex network, analysis of topological characteristics and feature extraction (Barry, 2005): texts can be modeled by using Complex Networks, so that different texts can be distinguished using the correlation between network parameters and text quality (Antiqueira et al., 2005). Textures, when modeled in such way, present their complex texture patterns represented by network connectivity. So, a feature vector based in traditional connectivity measurements allows texture characterization and classification (Thomas Chalumeau et al., 2006).

This work uses the Complex Network Theory in a similar approach to the cited works above. The focus is the shape boundary analysis and its identification, a feature of great importance in human communication.

2.1. Shape contour as a complex network

Literature presents various methods to analyze images and objects using the shape boundary. Most of them consider shape as a sequence of connected points where the order of these points expresses some meaning. Thus, the information of a shape contour can be described as a list S, $S = [s_1, s_2, \ldots, s_N]$, of N size where $s_i = (x_i, y_i)$ are discrete numerical values representing the coordinates of point i of the contour.

To apply complex networks theory to the problem, a graph representation of the shape contour is necessary (Fig. 1). A graph $G = (V, E)$ is built, where each point of the contour $s \in S$ corresponds to a vertex $v \in V$ in the graph G. A set of non-directed edges $E : V \times V$ binding each pair of vertices is also built. As in the work of Belongie et al. (2002), the set E is achieved by calculating the Euclidean distance between each pair of points of the contour:

$$d(s_i, s_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \quad (1)$$

Hence, the network is represented by the $N \times N$ weight matrix W

$$W(i,j) = w_{ij} = d(s_i, s_j) \quad (2)$$

normalized into the interval $[0, 1]$,

$$W = \frac{W}{\max_{w_{ij} \in W}}. \quad (3)$$

At this stage, each network vertex presents the same number of connections, i.e., a regular behavior. Nevertheless, this network does not present any relevant property for the proposed application, not even being a complex network. Following sections describe how to transform this network in a complex network as also the property of our interest.

2.2. Network degree

Degree (or *connectivity*) of a vertex v_i, $deg(v_i)$, is defined as being the number of edges in the network that are bound to v_i, i.e., the number of incident edges in v_i. Let $\partial v_i = \{v_j \in V | (v_i, v_j) \in E\}$ the set of neighbors of v_i, the degree of a vertex v_i is defined as

$$deg(v_i) = |\{e \in E | v_i \in e\}| = \left|\{v_j \in V | \{v_i, v_j\} \in E\}\right| = |\partial v_i|, \quad (4)$$

where $|A|$ denotes the cardinality (number of elements) of a set A (Wuchty and Stadler, 2003).

From degree distribution analysis, several measurements can be computed from a network. Three measurements considered for the proposed application are the *minimum degree*

$$Min(G) = \min_i deg(v_i), \quad (5)$$

the *Average Degree*

$$Av(G) = \sum_{v_i \in V} \frac{deg(v_i)}{|N|}, \quad (6)$$

and the *maximum degree*

$$Max(G) = \max_i deg(v_i). \quad (7)$$

2.3. Dynamic evolution

Different networks may present a large range in theirs characteristics. This makes the modeling of the dynamics of a complex network a difficult task. Moreover, a network characterization is not complete without considering the interaction between structural and dynamical aspects (Boccaletti et al., 2006).

Although modeling the dynamics of a complex network is a difficult task, additional information about structure and dynamic of complex networks can be yielded by applying a transformation over the original network and, in the sequel, by computing its properties (Costa et al., 2007). This transformation can be performed in many ways. Applying a threshold t over the edges E, in order to select E^*, $E^* \subseteq E$, so yielding a new network $G^* = (V, E^*)$ is a simple and straight approach. In this approach, each edge of E^* has a weight equal or smaller than t and this δ_t transformation is represented as

$$E^* = \delta_t(E) = \{e \in E | w(e) \leqslant t\}. \quad (8)$$

This δ_t transformation allows to study network properties at intermediate steps of its evolution, i.e., the properties of the sub-networks yielded as the maximum weight for its edges increases. By