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Perspective Article

Estimating depolarization with the Jones matrix quality factor

James N. Hilfiker [a,*], Jeffrey S. Hale [a], Craig M. Herzinger [a], Tom Tiwald [a], Nina Hong [a], Stefan Schöche [a], Hans Arwin [b]

[a] J.A. Woollam Co. Inc., 645 M Street, Lincoln, NE 68508, USA
[b] Department of Physics, Chemistry, and Biology, Linköping University, SE 581 83 Linköping, Sweden

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ABSTRACT

Mueller matrix (MM) measurements offer the ability to quantify the depolarization capability of a sample. Depolarization can be estimated using terms such as the depolarization index or the average degree of polarization. However, these calculations require measurement of the complete MM. We propose an alternate depolarization metric, termed the Jones matrix quality factor, Q_{JM}, which does not require the complete MM. This metric provides a measure of how close, in a least-squares sense, a Jones matrix can be found to the measured Mueller matrix. We demonstrate and compare the use of Q_{JM} to other traditional calculations of depolarization for both isotropic and anisotropic depolarizing samples; including non-uniform coatings, anisotropic crystal substrates, and beetle cuticles that exhibit both depolarization and circular diattenuation.

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1. Introduction

Spectroscopic ellipsometry measurements use polarized light to characterize thin films and bulk materials. During the measurement, the polarized measurement beam may transform into partially polarized light. This reduction in the degree of polarization for the measurement beam is referred to as depolarization. Depolarization is a feature of the sample or measurement caused by non-uniform or patterned films, finite bandwidth, angular beam spread, scattering, or a collection of multiple, incoherent beams such as from front and back of a thick substrate [1]. Correctly modelling these effects can improve accuracy of thin film characterization.

Mueller matrix (MM) measurements offer a complete description of the polarization-transformation of a sample or optic, including its depolarization capability. There are nine degrees of freedom within the MM associated with depolarization. Unfortunately, it is difficult to visualize whether a MM is depolarizing simply from examining its elements. To help, various single-valued metrics have been developed to estimate the depolarization capability from the full MM [2,3].

We propose an alternate depolarization metric, termed the Jones matrix quality factor, Q_{JM}. This term provides a measure of how close a best-fit Jones matrix is to the measured Mueller matrix. Since the Jones matrix is intrinsically non-depolarizing, the difference between the best-fit Jones matrix and the measured Mueller matrix is a figure of merit for the amount of depolarization. A key advantage of Q_{JM} is the ability to estimate depolarization even from an incomplete MM.

2. Theoretical background

For ellipsometry measurements of isotropic samples, it is common to estimate depolarization as [4]:

$$\%Depol = 100\left[1 - \left(N^2 + C^2 + S^2\right)\right] \tag{1}$$

where N, C, and S are elements of the normalized Mueller matrix defined as:

$$\mathbf{M}_{isotropic} = \begin{bmatrix} 1 & -N & 0 & 0 \\ -N & 1 & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix} \tag{2}$$

* Corresponding author.
E-mail address: jhilfiker@jawoollam.com (J.N. Hilfiker).

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The value of *%Depol* ranges from 0% for non-depolarizing samples to 100% for a completely depolarizing ellipsometry measurement. While only three elements of the normalized MM are required for this calculation, it is only valid for isotropic samples. Depolarization can be estimated for anisotropic, cross-polarizing samples, when the complete MM is measured, using terms such as the quadratic depolarization index, P_D, [2]:

$$P_D = \sqrt{\frac{\sum_{ij} m_{ij}^2 - m_{11}^2}{3m_{11}^2}} \quad (3)$$

where m_{ij} are the individual MM elements. The value of P_D ranges from 0 for completely depolarizing samples to 1 for non-depolarizing samples. Here we describe an alternate quantity called the Jones matrix quality factor, Q_{JM}. This term provides a measure of how close a best-fit Jones matrix is to the measured Mueller matrix. The calculation finds a normalized Jones matrix, **J**, that best corresponds to the measured normalized Mueller matrix, **M**, even if **M** does not correspond exactly to a Jones matrix. The conversion between Jones and Mueller matrices can be found in the review by Chipman [5]. To evaluate the closeness of **J** to **M**, the corresponding normalized Mueller matrix for **J** (referred to here as **N**) is calculated by minimizing the difference between **M** and **N**, and we define:

$$Q_{JM} = \sqrt{\frac{1}{x_{\exp} - x_{fit}} \sum_{ij} \left(m_{ij} - n_{ij}\right)^2} \quad (4)$$

with

$$m_{11} = n_{11} \equiv 1 \quad (5)$$

Here, x_{exp} is the number of measured MM values and x_{fit} is the number of real-valued Jones matrix parameters that were adjusted (real and imaginary parts count as separate fit parameters). All ellipsometer configurations we will discuss incorporate at least one compensator and $x_{fit} = 6$. Ellipsometers without at least one compensating element, such as rotating polarizer and rotating analyzer configurations, collect fewer MM elements and are incapable of characterizing depolarization.

If the measured MM is perfectly matched by an equivalent Jones matrix, then $Q_{JM} = 0$ and the MM does not depolarize. In fact, when $Q_{JM} = 0$ the MM is both non-depolarizing and physically realizable since it is matched perfectly by an equivalent Jones matrix. The presence of depolarization is indicated by $Q_{JM} > 0$. It should be noted that Q_{JM} is not an additional measurement parameter, but simply an indication of the depolarizing capability from the measured MM parameters. All information is contained within the MM parameters themselves.

We show MM measurements or simulated results for three instrument types: dual-rotating compensator ellipsometers (dual-RCE), polarizer-compensator-sample-analyzer (PCSA) and polarizer-sample-compensator-analyzer (PSCA) ellipsometers [6–8]. Equations (6)–(8) show the normalized MM elements that can be measured by each ellipsometer configuration.

$$\mathbf{M}_{Dual-RCE} = \begin{bmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad (6)$$

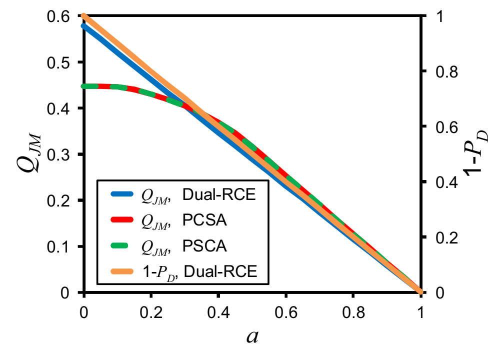

Fig. 1. Comparison of metrics for a uniform, partial depolarizer where a (Eqn. (9)) defines the diagonal MM elements.

$$\mathbf{M}_{PCSA} = \begin{bmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (7)$$

$$\mathbf{M}_{PSCA} = \begin{bmatrix} 1 & m_{12} & m_{13} & \cdot \\ m_{21} & m_{22} & m_{23} & \cdot \\ m_{31} & m_{32} & m_{33} & \cdot \\ m_{41} & m_{42} & m_{43} & \cdot \end{bmatrix} \quad (8)$$

While not specifically addressed here, similar results can be found for phase modulation ellipsometers, based entirely on the total number of MM elements measured, which depends on the number of phase modulators used and the optical configuration during measurement. Thus, the Q_{JM} calculation is related to the number of MM elements that are measured, and not specific to the ellipsometer technology that allows measurement of these elements.

As defined by Eqn. (4), the maximum range for Q_{JM} depends on the number of measured normalized MM elements. For PCSA and PSCA systems, $x_{exp} = 11$ and a fully depolarizing MM ($a = 0$ in Eqn. (9)) results in a maximum value of $Q_{JM} = 1/\sqrt{5} \sim 0.447$. When the full MM is measured, $x_{exp} = 15$ and the maximum value of $Q_{JM} = 1/\sqrt{3} \sim 0.577$ for fully depolarizing MM.

Fig. 1 compares values of Q_{JM} and 1-P_D for a uniform, partial depolarizer, represented as [9]:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \quad (9)$$

When the full MM is measured (dual-RCE), 1-P_D has the same shape as Q_{JM} although with different scaling. Q_{JM} is also calculated for PCSA and PSCA configurations, which result in a different shape and range.

The Q_{JM} metric can be rescaled to values between 0 and 1, but specific to certain ellipsometer configurations. For example, Q_{JM16} can be formulated to scale from 0 to 1 for measurements of the complete MM from dual-RCE instruments, with:

$$Q_{JM16} = \sqrt{3} \cdot Q_{JM} \quad (10)$$

3. Experimental

A dual-RCE instrument (Woollam RC2®) was used to measure the complete MM for energies from 0.73 eV to 6.46 eV (192 nm to 1690 nm). This instrument collects light on a silicon CCD for

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