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# A game-theoretic approach to sub-vertex registration☆

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ABSTRACT

Surface registration is a fundamental technique in computer vision. Typically, it consists of two basic steps: a coarse registration, followed by a fine registration. A novel game-theoretic matching (GTM) algorithm was proposed recently to directly obtain a fine surface registration in a single step. The main idea of GTM is to cast the selection of point correspondences in an evolutionary game framework. However, GTM fails easily due to the lack of correct correspondences if model surface is in low resolution. To tackle this problem, in this paper, we propose a game-theoretic approach to establish sub-vertex correspondences. A new way to construct the payoff function to solve one-to-many matches is introduced. The weight population after evolving from replicator dynamics is used to compute the corresponding sub-vertex. The effectiveness of our proposed method is verified by extensive experiments. Though comparing with GTM and state-of-the-art Super4PCS, our method is accurate, efficient and especially robust in extreme situations of high noise and low resolutions. Finally, the sensitivity and the limitations of our method are discussed.



## 1. Introduction

Surface registration is one of the key problems in computer vision. It is a fundamental technique for 3D surface reconstruction, 3D object recognition and augmented reality [1]. Because rigid registration is the foundation of more advanced non-rigid registration, it has attracted considerable research interest in recent years.

The goal of surface registration is to estimate a 3D rigid transformation between two surfaces, so that they can be placed together under a minimal distance measure. The transformation is denoted as a $3 \times 3$ rotation matrix $R$ and a $3 \times 1$ translation vector $t$. The two surfaces involving in registration can be simply represented as point sets. We name the matching target as model, while the matching source as data. An example of surface registration is shown in Fig. 1.

Surface registration is generally carried out in two steps: a coarse step and then a fine step. In both steps, correspondences are first found and then transformation is obtained in a closed-form solution. Since mature approaches such as the singular value decomposition (SVD) approach [2] or the quaternion-based approach [3] can be used to get transformation, establishing good correspondences is critical for a successful surface registration.

A recently proposed game-theoretic matching (GTM) [4] does not need coarse matching result as initialization. This algorithm outperforms most coarse registration algorithms and works almost equally well with traditional fine registration methods. One major disadvantage of this algorithm, however, is that correct correspondences cannot be found on surfaces of different resolutions. To overcome this difficulty, we propose herein a modified game-theoretic approach to sub-vertex registration. The sub-vertices can be seen as the interpolation of the original vertices. The weights are obtained through evolutionary game. As detailed in Section 4, our method can work well in extreme situations of high noise and low resolutions.

The focus of this paper is on establishing sub-vertex correspondences. The rest of the paper is organized as follows. After this introduction, we briefly review previous work as well as GTM in Sections 2 and 3. Then, the motivation behind our method and the details of it are presented in Section 4. Next, experimental results are given and compared with GTM and state-of-the-art technique in Section 5. Finally, some conclusions are drawn in Section 6.

## 2. Literature review

The goal of rigid surface registration is to minimize an objective function $\sum_{i=1}^{N} \|M_i - RD_i - t\|^2$ [3], where $\{M_i, i = 1, \ldots, m\}$ and $\{D_i, i = 1, \ldots, n\}$ are model and data vertices respectively and $N$ is

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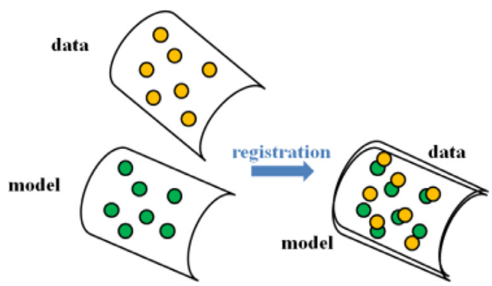


**Fig. 1.** An example of surface registration. Points on data and model surfaces are used for registration.

the number of established correspondences. The optimization is a two-stage problem of correspondence estimation and rigid transformation calculation.

As we have mentioned above, registration is usually implemented through the coarse and the fine step. Díez et al. [5] provided a good summary of coarse registration. Local feature based algorithms are often used [6–8] in the coarse step because of their convenience and low computational cost. Features specifically designed for GTM are presented in [9]. Guo et al. [10] gave a recent survey of existing local surface features for surface matching.

Fine registration is a refining process after the coarse step. The most popular algorithm in fine registration is Iterative Closest Point (ICP) [3] due to its simplicity and high accuracy. Bellekens et al. [11] offered a recent survey on ICP and its variants. ICP has been evolving over time and it has got improvements in speed, accuracy and robustness. Rusinkiewicz and Levoy [12] introduced a detailed classification of ICP variants. Since original objective function treats each correspondence equally, the influence of low-quality correspondences may result in poor registration. Therefore, assigning different weights for correspondences was proposed [12,13] and then the objective function changes into $\sum_{i=1}^{N} w_i \|M_i - RD_i - t\|^2$. The weight reflects the certainty of each established correspondence being a correct match. Recently, a sparse ICP [14] was proposed to robustly handle registration with outliers. Although ICP is popular, it heavily depends on the coarse registration results and easily converges to local minimum.

Global registration methods attract much attention recently since the two surfaces can be placed in arbitrary initial poses. Among them, game-theoretic matching (GTM) [4] is proposed to achieve the precision of fine registration without initial transformation estimation. It efficiently establishes robust one-to-one correspondences which mean that each data point takes at most one model point as its corresponding point. The state-of-the-art Super4PCS [15] finds transformation between two surfaces using coplanar sets of 4 points and can achieve outstanding results even when fine registration method fails.

Although GTM can achieve promising results in general occasions, the one-to-one matching limits its performance in the situation where surfaces have much different resolutions [16]. This is the main motivation for our method. To the best of our knowledge, this problem has not been explored before. In this paper, we proposed, implemented and verified the modified game-theoretic approach to sub-vertex registration in order to tackle the dilemma of GTM. Our method is also compared with both GTM and state-of-the-art Super4PCS.

## 3. A brief review of GTM

As our method is the modification of GTM, a brief review of GTM [4] is necessary.

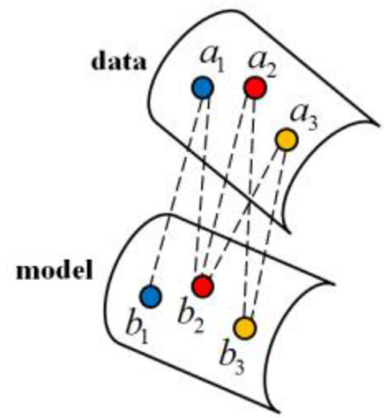


**Fig. 2.** An illustration of game-theoretic matching (GTM).

### 3.1. Game-theoretic matching

In the two-player's non-cooperative game, $S = \{1, \ldots, n\}$ is a set of available pure strategies and $\Pi : S \times S \to R$ is a payoff matrix, where $\pi_{ij} = \Pi(i, j)$ indicates the benefit that a player playing strategy $i$ gains when another player adopting strategy $j$. A mixed strategy is a probability distribution $\mathbf{x} \in \Delta$ denoted as a column vector over the strategies $S$, where $\Delta$ is defined in Eq. (1). $x_i$ is the distribution proportion of strategy $i$ and it is also the probability that the player will choose that strategy.

$$\Delta = \left\{ \mathbf{x} : x_i \geq 0, \forall i \in 1 \ldots n, and \sum_{i=1}^{n} x_i = 1 \right\} \tag{1}$$

The expected payoff obtained by a player playing mixed strategy $\mathbf{y} \in \Delta$ against another player adopting $\mathbf{x} \in \Delta$ is $\mathbf{y}^T \Pi \mathbf{x}$. The most important concept in game theory is Nash equilibria. A strategy pair $(\mathbf{x}, \mathbf{y})$ is a Nash equilibrium if $\mathbf{x}$ is the best reply to $\mathbf{y}$, and at the same time, $\mathbf{y}$ is the best reply to $\mathbf{x}$. Since we only consider symmetric games in which the two players are indistinguishable, only $(\mathbf{x}, \mathbf{x})$ are of interest [17]. Therefore, a strategy $\mathbf{x}$ is a Nash equilibrium if it is the best reply to itself. This means $\forall \mathbf{y} \in \Delta, \mathbf{x}^T \Pi \mathbf{x} \geq \mathbf{y}^T \Pi \mathbf{x}$. Further, a strategy $\mathbf{x}$ is called an evolutionary stable strategy (ESS) if it is first a Nash equilibrium and then robust to a small perturbation of itself [18]. As the expected payoff of the entire population is $\mathbf{x}^T \Pi \mathbf{x}$ [17], evolutionary game can be seen as an optimization problem defined in Eq. (2).

$$\max_{\mathbf{x} \geq 0} \mathbf{x}^T \Pi \mathbf{x}, s.t. \mathbf{x}^T \mathbf{1} = 1 \tag{2}$$

We use Fig. 2 to illustrate the process of GTM. The dashed lines represent the possible matching candidates. Imagine two players participating in the game and they pick up matching candidates $(a_1, b_1)$ and $(a_2, b_2)$ respectively. If the two candidates are compatible with each other, then both players receive a high score, otherwise the score will be low. The two candidates are compatible if they satisfy the rigidity constraint. In this example, $(a_1, b_1)$ and $(a_2, b_2)$ are compatible if the Euclidean distances $\|a_1 - a_2\|$ and $\|b_1 - b_2\|$ are equal. In order to gain high scores, each player tends to pick matching candidates that are compatible with the other player's choice. For more knowledge of GTM, please refer to [16] and [19].

### 3.2. Matching candidates, payoff function and evolution

Matching candidates can be established between any two vertices on data and model surfaces, which are quite a lot. They will definitely take large memory size while finally most of them will