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Detection of the non-topology preservation of Ma's 3D surface-thinning algorithm, by the use of P-simple points

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Abstract

Recently, Wang and Basu [Wang, T., Basu, A., 2007. A note on 'a fully parallel 3D thinning algorithm and its applications'. Pattern Recognition Lett. 28 (4), 501–506] have written a paper in which they claim that Ma and Sonka's 3D thinning algorithm [Ma, C., Sonka, M., 1996. A fully parallel 3D thinning algorithm and its applications. Computer Vision and Image Understanding 64 (3), 420–433] does not preserve topology. As they highlight in their paper, a counterexample has been given in Lohou's thesis [Lohou, C., 2001. Contribution à l'analyse topologique des images: étude d'algorithmes de squelettisation pour images 2D et 3D selon une approche topologie digitale ou topologie discrète. Ph.D. thesis, Univ. de Marne-la-Vallée, France]. In fact, the previous Ma's algorithm [Ma, C., 1995. A 3D fully parallel thinning algorithm for generating medial faces. Pattern Recognition Lett. 16, 83–87] does not preserve topology. The goal of this paper is to show how P-simple points have guided us towards a proof that Ma's algorithm does not always preserve topology.

Keywords: Digital topology; 3D parallel thinning algorithm; Topology preservation

1. Introduction

Some medical or graphical applications require the transformation of objects while preserving their topology. That leads to the well-known notion of a simple point: a point in a binary image is said to be *simple* if its deletion from the image "preserves the topology" (Kong and Rosenfeld, 1989). Thinning algorithms are usually designed as processes which remove simple points and obey several other criteria. In fact, during the thinning process, certain simple points are kept in order to preserve some geometrical properties of the object. Such points are called *end points*. For the 3D case, we can define two different kinds of end points: curve-end points and surface-end points (Borgefors et al., 1999). A thinning process which preserves curve-end (resp. surface-end) points is called a *curve* (resp. a *surface*) *thinning algorithm*.

A major problem which arises when designing thinning algorithms is that the simultaneous removal of simple points may change the topology of an object. This is the case, for example, of a 2-voxel thick ribbon: if we delete in parallel all simple points of such an object, it will disappear. To solve this problem, three different solutions may be considered:

- either points which may be deleted must match at least one amongst several given $3 \times 3 \times 3$ masks or templates. Note that the templates are proposed in such a way that the algorithm based on these templates preserves the topology (Palágyi and Kuba, 1998a,b),
- or it is allowed to access to a neighborhood greater than the $3 \times 3 \times 3$ neighborhood centered around a considered point. Such a strategy may lead to fully parallel thinning algorithms (Ma, 1995; Ma and Sonka, 1996; Manzanera et al., 1999),
- or another class of simple point must be found in such a way that if we delete in parallel such points, then the

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topology is preserved. This is what has been accomplished by the introduction of P-simple points (Bertrand, 1995a). In fact, this notion leads to different thinning schemes (Lohou and Bertrand, 2004, 2005, 2007).

We have the property that any algorithm removing only subsets composed solely of P-simple points is guaranteed to keep the topology unchanged. Let us consider a given algorithm A. Let X be any subset of $\mathbb{Z}^3$, we define the set P as the set of points of X which are considered as deletable by A. Then, we try to prove that any point of P is a P-simple point. If it is the case, then A is ensured to preserve topology. Elsewhere, that means we have found an object which contains a point x which is deletable by A, and such that x is not P-simple. That does not imply that the algorithm does not preserve topology (Lohou, 2001), nevertheless, it may be interesting to deeper examinate this object: perhaps, A does not preserve topology when the point x is deleted. This is what we have obtained when we checked the soundness of Ma's algorithm (Lohou, 2001). The goal of this paper is to show how, thanks to the notion of P-simple point, we have found an object which has helped us to prove that Ma's algorithm does not always preserve the topology.

2. Basic notions of digital topology

2.1. Neighborhoods, connected components and holes

A point $x \in \mathbb{Z}^3$ is defined by (x_1, x_2, x_3) with $x_i \in \mathbb{Z}$. We consider the three neighborhoods: $N_{26}(x) = \{x' \in \mathbb{Z}^3 : Max[|x_1 - x'_1|, |x_2 - x'_2|, |x_3 - x'_3|] \leqslant 1\}$, $N_6(x) = \{x' \in \mathbb{Z}^3 : |x_1 - x'_1| + |x_2 - x'_2| + |x_3 - x'_3| \leqslant 1\}$, and $N_{18}(x) = \{x' \in \mathbb{Z}^3\colon |x_1 - x'_1| + |x_2 - x'_2| + |x_3 - x'_3| \leqslant 2\} \cap N_{26}(x)$. We define $N^*_n(x) = N_n(x) \setminus \{x\}$. We call, respectively, 6-, 18-, 26-*neighbors* of x the points of $N^*_6(x)$, $N^*_{18}(x) \setminus N^*_6(x)$, $N^*_{26}(x) \setminus N^*_{18}(x)$, such points are represented in Fig. 1a.

The 6-neighbors of x determine six major directions (Fig. 1b): Up, Down, North, South, West, East; respectively, denoted by U, D, N, S, W and E. Let Dir denote one of these six directions. The point in $N^*_6(x)$ along the direction Dir is called the *Dir-neighbor* of x and is denoted by $Dir(x)$. Let $X \subseteq \mathbb{Z}^3$. If the Dir-neighbor of x belongs to

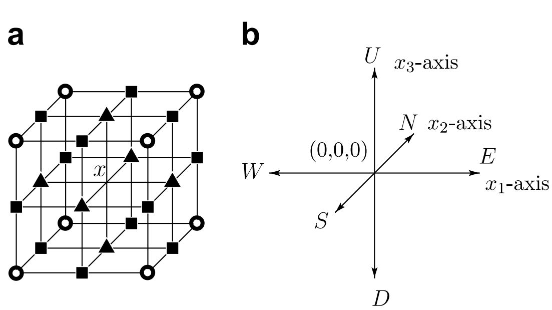

Fig. 1. (a) The 6-, 18-, and 26-neighbors of x are, respectively, represented by black triangles, black squares, and black circles, (b) the six major directions.

$\overline{X}$ (the complement of X in $\mathbb{Z}^3$), then x is said to be a *Dir-border point*. The points belonging to X (resp. $\overline{X}$) are called *black points* (resp. *white points*).

Two points x and y are said to be *n-adjacent* if $y \in N^*_n(x)$ ($n = 6, 18, 26$). An *n-path* is a sequence of points $x_0, \ldots, x_k$, with x_i n-adjacent to x_{i-1} for any $1 \leqslant i \leqslant k$. If $x_0 = x_k$, the path is *closed*. Let $X \subseteq \mathbb{Z}^3$. Two points $x \in X$ and $y \in X$ are *n-connected in* X if they can be linked by an n-path included in X. The equivalence classes relative to this relation are the *n-connected components of* X. In order to have a correspondence between the topology of X and that of $\overline{X}$, we have to consider two different kinds of adjacency for X and for $\overline{X}$ (Kong and Rosenfeld, 1989): if we use an n-adjacency for X, we have to use another $\overline{n}$-adjacency for $\overline{X}$. In this paper, we only consider $(n, \overline{n}) = (26, 6)$.

Let $X \subset \mathbb{Z}^3$ and $x \in X$. A *hole* (sometimes called a *tunnel*, see Kong and Rosenfeld, 1989; Ma, 1994) in X is detected when there exists a closed path in X which cannot be deformed in X into a single point. In $\mathbb{Z}^3$, a deformation is a sequence of elementary deformations such that a closed path Γ' is an elementary deformation of a closed path Γ if Γ and Γ' are the same excepted in a unit cube (Kong and Rosenfeld, 1989; Bertrand, 1994).

2.2. Simple points and topological numbers

Let $X \subseteq \mathbb{Z}^3$. A point $x \in X$ is said to be *simple* if its removal does not "change the topology" of the image, in the sense that there is a one-to-one correspondence between the components, the holes of X (resp. $\overline{X}$) and the components, the holes of $X \setminus \{x\}$ (resp. $\overline{X} \cup \{x\}$), see Kong (1989) for a precise definition.

The set composed of all n-connected components of X which are n-adjacent to a point x is denoted by $\mathscr{C}^x_n(X)$. Let $\#X$ denote the number of elements which belong to X. The *topological numbers* relative to X and x are the two numbers (Bertrand, 1994): $T_6(x, X) = \#\mathscr{C}^x_6[N^*_{18}(x) \cap X]$ and $T_{26}(x, X) = \#\mathscr{C}^x_{26}[N^*_{26}(x) \cap X]$. These numbers lead to a very concise characterization of 3D simple points (Bertrand and Malandain, 1994): $x \in X$ is simple for X if and only if $T_{26}(x, X) = 1$ and $T_6(x, \overline{X}) = 1$. Let us consider the two first configurations depicted in Fig. 2. In (a), for example, both two points y and z belong to $N^*_{18}(x) \cap \overline{X}$ but there is no 6-path of white points included in $N^*_{18}(x) \cap \overline{X}$ which joins them; w belongs to $N^*_{18}(x) \cap \overline{X}$ but is not 6-adjacent to x, thus $(T_{26}(x, X), T_6(x, \overline{X})) = (1, 2)$; therefore x is not simple. In (b), there is a single black 26-connected component in $N^*_{26}(x) \cap X$, w does not belong to $N^*_{18}(x) \cap \overline{X}$, thus $(T_{26}(x, X), T_6(x, \overline{X})) = (1, 1)$; therefore x is simple.

2.3. Simple sets and minimal non-simple sets

Let X be a subset of $\mathbb{Z}^3$. A subset $S \subset X$ is a *simple set* of X if the points of S can be arranged in a sequence $S = \{x_1, \ldots, x_k\}$ in such a way that x_1 is simple for X and x_i is simple for the subset $X \setminus \{x_1, \ldots, x_{i-1}\}$, for $i = 2, \ldots, k$. A subset $S \subset X$ is a *minimal non-simple set* if