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Wavelet-based fuzzy multiphase image segmentation method ☆

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ABSTRACT

This letter proposes a novel fuzzy multiphase image segmentation model. In the model, we introduce wavelet based regularization on the membership functions which are used as indicators of different regions. By using principal component analysis (PCA) features data as descriptors, the proposed model can segment texture and natural images. To efficiently solve the model, we formulate a fast iterative shrinkage algorithm for multiphase image segmentation. Experimental results show that the proposed method achieves better segmentation results compared with some other classical variational segmentation methods.

1. Introduction

Image segmentation is a fundamental problem in image processing and computer vision. Image segmentation refers to the process of partitioning an image into several regions or locating objects and boundaries. Many variational segmentation models have been developed for this task. These models can be roughly classified into the following three categories: level-set-based methods [1,2], phase-field-based methods [3], and fuzzy-based methods [4–9]. To represent different regions, the first two kinds of methods use level set functions and sinusoidal potential functions respectively, which automatically avoid overlap or vacuum of different regions (hard methods). The third kind of method uses overlapping different membership functions to indicate different regions (soft methods). Different from the hard methods, in the soft methods, fuzzy membership functions are introduced to measure the association degree of each pixel to all image regions. The probability is represented by fuzzy membership functions valued in [0, 1]. Recently, more and more authors pay attention to fuzzy-based segmentation methods because of the flexibility of fuzzy membership functions. Various two-phase fuzzy segmentation models have been proposed [4,5]. By adding some constraints, Li et al. [6] generalize the two-phase fuzzy region competition to a multiphase segmentation model based on the piecewise constant function. Later, Li and Ng [7] use the nonparametric technique kernel density estimation in the multiphase fuzzy region competition framework to segment texture images. Based on a nonconvex regularization term, Han et al. [8] propose a variational model for the fuzzy multiphase image segmentation. By taking into account both spatial and frequency information in data fidelity term, Choy et al. [9] present a multiphase image segmentation model within fuzzy region competition frame. The use of frequency data improves the overall segmentation result. In these works, soft methods show better performance and higher efficiency than hard methods.

We note that most of the above are TV-based methods. Motivated by the region competition [6], this letter proposes a novel wavelet-based fuzzy multiphase image segmentation model which makes use of the advantages of wavelet decompositions. Firstly, we briefly review and analyze the model in [6]. Let Ω be the image domain and f be a given gray level image. According to the features of f, the goal of image segmentation is to partition Ω into N regions $(\Omega_i)_{i=1,2,\ldots,N}$. The multiphase segmentation model in [6] is equivalent to the following minimization problem

$$\min_{0\le I_i\le 1}\left\{\begin{array}{l} F=\displaystyle\sum_{i=1}^{N}\int_\Omega |\nabla I_i(x)|\,\mathrm{d}x+\lambda\sum_{i=1}^{N}\int_\Omega (f-c_i)^2 I_i(x)\,\mathrm{d}x \\ \quad +\dfrac{\eta}{2}\displaystyle\int_\Omega\left(\sum_{i=1}^{N} I_i(x)-1\right)^2\mathrm{d}x \end{array}\right\}, \qquad (1)$$

where λ and η are fixed positive parameters. I_i $(i = 1, 2, \ldots, N)$ in [0, 1] is the i-th membership function which indicates the i-th region Ω_i $(i = 1, 2, \ldots, N)$. The first term is the regularization term which characterizes some features of the desired solution $\hat{I}_i$. $\int_\Omega |\nabla I_i(x)|\,\mathrm{d}x$ is the BV-seminorm of I_i. It is also referred to as the total variation (TV) of I_i. The second term is a data fidelity term which is defined to evaluate the performance of the label assignment at each partition Ω_i. The parameter c_i is the mean intensity value of f on Ω_i. That is to say, the model assumes that the given image f is a piecewise constant function. The third term is a quadratic penalty term. An alternate minimization method is adopted to find the optimal solution $\hat{I}_i$. Experimental results in [6] have shown very promising results. However,

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there are four main drawbacks in the model (1). (i) The total variation (TV) is used as a convex regularization term to preserve geometrical structure of each membership function. TV regularization has a good capability of preserving edges of uniform but it does strongly smooth (oversmoothing edges of disjoint regions), and may even destroy small scale structures with high curvature edges. (ii) Though the fast Chambolle's dual projection algorithm [10] can be adopted, the algorithm in [6] requires inner iteration in the whole optimization process. (iii) For the piecewise constant function assumption, the model (1) cannot be used to partition images containing textures or general natural images. (iv) To ensure the constraints $\sum_{i=1}^{N} I_i \approx 1$, the penalty parameter η should be set very large which will lead to numerical instability.

2. Contributions

Wavelet shrinkage is a well-established method for image denoising. In this letter we formulate a novel multiphase image segmentation method based on wavelet iterative shrinkage. The results of this letter can be extended to a number of image segmentation models involving TV regularization.

Firstly, this letter presents a novel wavelet-based fuzzy multiphase image segmentation model. In our model, a wavelet regularization term on the membership functions is introduced. We extend standard total variation regularization techniques, recently developed for fuzzy image segmentation, to wavelet-based regularization techniques which can preserve edges and shape of segmentation results better. Wavelet transforms based techniques have been successfully applied in various image processing problems, such as image restoration, image super-resolution, image fusion, image quality assessment, image feature extraction, etc. We note that wavelets are usually used in extracting feature or feature selection in most wavelet based segmentation methods. In this letter, we discuss our wavelet based segmentation method in the variational framework. The most important advantage of the variation-based segmentation methods is that they can be solved by more efficient algorithms. Secondly, using PCA features to the proposed wavelet based image segmentation model, our model generalizes classical fuzzy segmentation models which do not deal with the problem of partitioning texture and natural images. Thirdly, an alternate minimization method is designed to find the optimal solution by using convex optimization tool. The proposed algorithm integrates the split Bregman and fuzzy segmentation method.

The proposed method is directly compatible with most types of TV-based fuzzy multiphase image segmentation implementation. In fact, the proposed algorithm is a special type of wavelet iterative shrinkage algorithm, which is very simple and easy to use. Moreover, the proposed algorithm is easy to extend to use other multiscale analysis tools such as curvelets, bandlets etc.

3. The proposed model and algorithm

3.1. The proposed model

Images usually consist of different components. They may contain smooth structures and complex textures. The performances of different segmentation models not only depend on the models themselves but also vary with the images to be segmented. The better the images comply with the assumptions of a model, the more effective results can be obtained from the model. Both the LS model and the TVR model, of course, can be simply applied to partition texture images. However, texture images do not rigidly comply with the assumptions of these models. Therefore, their corresponding segmentation results are usually not quite satisfactory.

In this letter, we use the way of abstracting pixel descriptors proposed by Rao et al. in [11]. A $m \times m$ cut-off window is extracted around each pixel. The $m \times m$ pixels of an image patch are arranged in a m^2-dimensional column vector. We reduce the dimension of each vector by projecting all of the vectors onto their first K principal components by using the PCA method. We call this kind of descriptor as the PCA descriptor. In the experiments, we set $m = 5$, $K = 8$. Han et al. [8] also use the same method. After we obtain the PCA descriptor, we use the notation $f_j(x)$ $(j = 1, 2, \ldots, K)$ to represent the j-th principal component of the projected image. Our segmentation model is proposed as follows

$$\min_{I_i} \left\{ J = \sum_{i=1}^{N} \sum_{\gamma\in\Gamma} \left|(I_i)_\gamma\right| + \lambda \sum_{i=1}^{N} \sum_{j=1}^{K} \int_\Omega (f_j - c_{ij})^2 I_i(x)\,\mathrm{d}x \right\}.$$

$$\text{s.t. I.} \sum_{i=1}^{N} I_i(x) = 1, \text{ II. } 0 \le I_i(x) \le 1, \quad i = 1, 2, \ldots, N \tag{2}$$

The first term is a regularization term, where I_i $(i = 1, 2, \ldots, N)$ is a fuzzy membership function, $\sum_{\gamma\in\Gamma} |(I_i)_\gamma|$ is l_1 norm of wavelet decomposition coefficients $(I_i)_\gamma$ of I_i $(i = 1, 2, \ldots, N)$, and Γ denotes the set of indices of wavelet coefficients. The second term is a data fidelity term, where λ is a regularization parameter which controls the trade-off between the regularization term and data fidelity term. f_j $(j = 1, 2, \ldots, K)$ is the j-th PCA feature component of the given image f. c_{ij} is the mean intensity value of f_j *on* Ω_i $(i = 1, 2, \ldots, N, j = 1, 2, \ldots, K)$. Other forms of data fidelity term also can be similarly discussed in our framework, such as parameterization models [4].

Note that we replace the BV-seminorm in model (1) by the l_1-norm of wavelet coefficients. One important fact is that the sparseness of a wavelet expansion is equivalent to smoothness measure in a Besov space. Specifically, the l_1-norm of wavelet coefficients $(I_i)_\gamma$ is equivalent to the norm in Besov space $B_1^{1,1}(\Omega)$, which is a subspace of BV(Ω) [12,13]. In model (2), we introduce the PCA descriptors into the data fidelity term. For a homogeneous texture, the PCA descriptor can be modeled by a Gaussian distribution. As mentioned in [11], over all distributions with the same variance, the Gaussian distribution is the worst case for compression and has the highest rate-distortion. By using the Gaussian distribution to model a homogeneous texture, an upper bound can be obtained for the true coding length of Gaussian Mesh Markov Model. Based on this point, it is reasonable to use the Gaussian distribution to model the PCA descriptor in a homogeneous texture region. From the probability viewpoint, the data fidelity term is derived from the assumption that the descriptors (gray/color intensities) for each pixel in the whole/local part of a homogeneous region in an image is sampled from an independent and identical distribution. When the Gaussian distribution is used, the descriptors for all pixels in the image are assumed to be piecewise constants [1] or local piecewise constants [2,6].

The proposed model has three main advantages. Firstly, the wavelet regularization term performs better than TV regularization term. It protects edges from oversmoothing which is a common drawback of TV regularization term. Secondly, the multiscale geometric analysis tool can preserve geometric shape of the segmentation regions better. Thirdly, by introducing the PCA features, the proposed method can partition texture images.

3.2. The algorithm

In this subsection, we design a fast iterative shrinkage algorithm for the model (2). For simplicity, we drop the variable x in some places. In iterative process, we project each membership function $I_i (i = 1, 2, \ldots, N)$ onto the interval [0, 1] by using $I_i^{k+1} = \min\{\max\{I_i^{k+1}, 0\}, 1\}, i = 1, 2, \ldots, N$, which ensures the optimal solution to satisfy the condition (II) in the model (2). To satisfy condition