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Efficient classification using the Euler characteristic ☆

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ABSTRACT

This paper introduces an object descriptor for classification based on the Euler characteristic of subsets created by thresholding a function at multiple levels (sub-level filtration). We demonstrate the effectiveness of this basic topological invariant of sets, the Euler characteristic, and use it to compute descriptors in two different domains – images and 3D mesh surfaces. The descriptors used as input to linear SVMs achieve state of the art classification results on various public data sets. Moreover, these descriptors are extremely fast to compute. We present linear time methods to calculate the Euler characteristic for multiple threshold values and to compute the Euler characteristic in a sliding window.

1. Introduction

Supervised object classification entails two main elements – features (descriptors) and learning algorithms. In recent years, much effort has been invested in developing features that yield good classification. Different features are developed for different domains such as images and 3D objects. Some features are even object specific, e.g. faces or texture.

For good classification, features should be rich, descriptive and discriminative, and on the other hand, invariant to different transformations and robust enough to allow intra-class variation. The focus in recent years shifted from *global features* that describe the object as a whole, to statistical descriptors of *local features*. The statistical descriptor of low-dimensional local features is simply their distribution (e.g. color histogram). For more complex local features (e.g. SIFT), a *Bag-of-Words* (BoW) scheme is used.

The main criticism for statistical descriptors of local features is the loss of all spatial information: "because these methods disregard all information about the spatial layout of the features, they have severely limited descriptive ability" [14]. For example, in one object class, the different values of the local feature might be evenly distributed, and in another class, the different values are clustered. Several approaches for putting the local features into some spatial (or spatio-temporal) context have been suggested. Some examples are spatial pyramids [14] and hierarchical neighborhood features [12].

This paper presents a new descriptor for supervised classification, which is based on simple local features, but instead of using their distribution, we propose to threshold the feature at multiple levels and calculate the Euler characteristic (EC) values of the resulting subsets of the domain. The vector of Euler Numbers – the Euler Characteristic Graph (or EC Graph) is then used as an object descriptor.

The EC is a global topological invariant which in the case of a two-dimensional set is the number of connected components minus the number of holes. The EC Graph feature therefore encodes information about the spatial distribution of the local property, information which is missing in many statistical descriptors. One of the nice algorithmic properties of EC Graph is that it can be easily computed by counting local elements. The EC is invariant to all topological transformations, including rotation and scale. Section 6.2.2 shows that the EC Graph descriptor has better invariance to image transformations than the global distribution descriptor [21].

We evaluate the performance of the EC Graph descriptor using images and 3D mesh objects, based on different local properties. In both domains we use publicly available object classification datasets and show that the EC Graph feature achieves state of the art results at very low computation time.

The rest of the paper is organized as follows: In Section 2 we review existing methods for calculating the EC and for its use as a descriptor. The EC Graph descriptor is introduced in Section 3. Sections 4 and 5 present our efficient methods for calculating the EC Graph on the entire domain and in a sliding window. In Section 6 we provide experimental results for the efficient calculation methods and evaluate the performance of the EC Graph feature for classification. We provide our conclusions in Section 7.

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The contributions of this paper:

- EC Graph – A new descriptor for supervised classification based on the Euler characteristic of simple local features.
- An algorithm for efficient calculation of the EC Graph for multiple thresholds, for regular and non-regular domains.
- An algorithm for efficient calculation of the EC Graph in a sliding window (can be used for object detection).
- Evaluation of the EC Graph feature for classification of images and 3D mesh objects.

2. Related work

2.1. The Euler characteristic

The Euler characteristic (or Euler number) dates back to Leonhard Euler (1707–1783) who observed that in simple polyhedra, $V - E + F = 2$ i.e. the number of vertices minus the number of edges plus the number of faces always equals 2. This was later generalized to the topological invariant: $\chi = V - E + F$, for any object constructed from 0,1 and 2-dimensional cells (vertices, edges and faces). In general, the EC χ of a set S is equal to the alternating sum of the cardinalities of the open k-dimensional cells in any partition of S.

It is important to note that the Euler characteristic of an object is independent of the cell-decomposition (also termed *triangulation*) being used. An example for a cell-decomposition is a uniform square lattice, in which a 2D planar set (e.g. an image) is constructed of open squares (pixels), their edges and corners. Another example is a triangle-mesh representing a surface in 3D, which is constructed from triangular faces, edges and vertices. In addition, it should be noted that the Euler characteristic is additive, obeying the inclusion–exclusion principle.

2.2. The Euler characteristic as an object descriptor

Several papers suggest using the Euler characteristic of a binary image for recognition or classification. For example, Anagnostopoulos et al. [2] use the EC of a binary image of a license plate for OCR. Mery et al. [16] suggest using the EC of corn tortillas to evaluate their quality (a high-quality tortilla is expected to have one connected component and no holes). In both papers, a single Euler number is calculated for a binary image that was created by segmenting the original image.

The concept of calculating the EC of subsets of the domain defined by multiple threshold values of a density function over the domain is suggested by Worsley [20]. The resulting graph is manually compared to the expected EC graph of a proposed stochastic model [1].

Huber et al. [10,22] use the Minkowski Functionals (MFs) of grayscale images at multiple threshold levels for classification. In both cases, the EC is calculated on the original grayscale image. We propose a more general descriptor, which is based on the EC graph of different local features instead of using the original image. We will show that this approach yields much better results and is applicable to different domains and not just to images.

2.3. Calculating the Euler characteristic

The Euler characteristic of a binary image on a continuous 2D plane and on a lattice (a discretized image) is defined by Gray [8]. Over the years, several methods were suggested for efficient calculation of the EC of a binary image (an image with pixel values of '0' and '1'). Gray [8] proposes a calculation method based on counting 2×2 pixel patterns (in case of a square lattice). Bribiesca [3] proposes a method based on the Contact Perimeter (length of edges adjacent to two pixels).

For grayscale images, simply repeating the EC calculation for multiple threshold levels will result in $O(NT)$ time complexity, where N is the number of pixels in the image and T is the number of threshold values. Snidaro and Foresti [19] and Conaire [5] propose a method that operates at $O(N + T)$, but since it is based on Gray's 2×2 pixel pattern, it is applicable to regular grids only. The method we propose has a time complexity of $O(N + T)$ as well, and is applicable also to non-regular domains, such as 3D mesh surfaces.

The Minkowski Functionals (which include the EC) are suggested as a feature for texture analysis, an application that requires calculating the MFs in multiple image sub-windows [15]. The authors describe a bias in the MFs calculation and propose an approximated solution by averaging MF values calculated using 8 different traversing directions. In Section 5 we propose a closed and efficient solution for calculating the EC in multiple sub-windows. Our method is based on triangulation (counting vertices, edges and faces) and on a modified Integral Image [6] calculation.

3. The Euler Characteristic Graph feature

Fig. 1 demonstrates the EC Graph feature for a two-dimensional image. A more formal definition is provided in the next section. The input image shown in Fig. 1(a) was generated by applying a Gaussian filter on a random image. Fig. 1(b)–(f) show how the original object is segmented by thresholding the input. As we increase the threshold, holes (shown in black) begin to appear and χ becomes negative. When we continue raising the threshold, the segmented object begins breaking up into components. χ will reach 0 again when the number of components is equal to the number of holes (Fig. 1(d)). χ continues to rise as we increase the threshold, until the object consists of many separate 'islands'. As we continue increasing the threshold, the islands will start to disappear and χ will move towards 0 again. Fig. 1(g) shows the resulting EC Graph.

4. Efficient calculation of the Euler Characteristic Graph

A set $X \subset \mathbb{R}^n$ can be represented, non uniquely, as a union of open k-cells (open elements of dimension $k, k \leqslant n$):

$$X = \bigcup_{k=0}^{n} \bigcup_{i=1}^{M(k)} x_i^{(k)} \tag{1}$$

where $x_i^{(k)}$ is an open cell of dimension k and $M(k)$ is the number of k-cells. The Euler characteristic χ of the set X is:

$$\chi = \sum_{k=0}^{n} (-1)^k M(k) \tag{2}$$

In some cases, as in most applications discussed in this paper, we need to find the EC of a subset S of the original domain, defined by a function over the domain. S can be represented as a union of open k-cells, as defined by (1). In this section we assume that S itself is closed.

We first discuss the case of a binary function: Let $f_d(x^{(d)}) \in \{0, 1\}$ be a binary function defined over the open cells of the highest dimension d of which X is constructed. For example, if X is a mesh surface in $\mathbb{R}^3$, f will be defined over the triangular faces ($d = 2$). The functions over the lower-dimension elements are not given as an input and are derived from f_d. We define $f_k(x^{(k)}), k < d$ to be 1 for all k-cells on the boundary of "1" d-cells, in which $f_d(x^{(d)}) = 1$:

$$f_k(x^{(k)}) = \mathbf{1}_{[\exists x^{(d)} \in N_d(x^{(k)}) : f_d(x^{(d)}) = 1]} \tag{3}$$