



Dynamic contact interactions of fractal surfaces



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ABSTRACT

Roughness parameters and material properties have significant influence on the static and dynamic properties of a rough surface. In the present paper, fractal surface is generated using the modified two-variable Weierstrass-Mandelbrot function in MATLAB and the same is imported to ANSYS to construct the finite element model of the rough surface. The force-deflection relationship between the deformable rough fractal surface and a contacting rigid flat is studied by finite element analysis. For the dynamic analysis, the contacting system is represented by a single degree of freedom spring mass-damper-system. The static force-normal displacement relationship obtained from FE analysis is used to determine the dynamic characteristics of the rough surface for free, as well as for forced damped vibration using numerical methods. The influence of fractal surface parameters and the material properties on the dynamics of the rough surface is also analyzed. The system exhibits softening property for linear elastic surface and the softening nature increases with rougher topography. The softening nature of the system increases with increase in tangent modulus value. Above a certain value of yield strength the nature of the frequency response curve is observed to change its nature from softening to hardening.

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1. Introduction

A variety of mechanical elements, such as gear, cam-follower mechanism, rolling element bearings have dynamic contact at their contact surfaces. It is also well known that all surfaces are inherently associated with roughness. So, it can be inferred that vibration (or dynamics) at the contacting rough surface has major influence on the fatigue and wear performance of such components. Hence the study of dynamic contact interactions at rough surfaces has always been an area of research interest. The modeling of the rough surface during the dynamic analysis can be done using fractal geometry which was first introduced by Mandelbrot [1–4]. Since a rough surface is proved to be a random non-stationary process [5] due to its scale dependent nature, it can be characterized by fractal geometry. The applicability of fractal geometry as an efficient model of rough surface was presented by several researchers [6–8]. The equation for generating fractal surface, i.e. the Weierstrass–Mandelbrot equation was modified by Berry and Lewis [9] and Ausloos and Berman [10]. Few general distribution functions involving fractal parameters and the corresponding contact model were presented [11,12]. It was also found that the fractal roughness parameters are dependent upon the mean separation of

two contacting surfaces [13,14]. Analytical approach on the real contact area and deformation of elastic and elastic-plastic contact of fractal surfaces was carried out by Yan and Komvopoulos [15]. Kogut and Jackson [16] conducted a comparison between contact mechanics results obtained with statistical and fractal approaches to characterize surface topography. Several researchers have undertaken the study of normal contact stiffness and contact area of fractal surfaces in elastic and elastic-plastic regime [17–22]. In the last two decades, Finite Element Analysis has evolved as an efficient tool for contact analysis. Hyun et al. [23] and Pei et al. [24] adopted a 3D finite element analysis for elastic and elasto-plastic contact between rough surfaces with a range of self-affine fractal scaling behaviour. Influences of material and surface parameters on elastic and elastic-plastic contact interactions was analyzed through finite element analysis by several researchers [25–28]. In most of research works on fractal contact analysis available in the literature, the contact is assumed to be equivalently modeled rough to flat [8,15,19,26] or rough to spherical contact [25]. But analysis with rough to rough contact is found to be comparatively less in number. In recent years some researches have taken up contact analysis for rough to rough contact interfaces as the domain of their work [29–31].

It has been seen that when it comes to dynamic contact interactions between rough surfaces, Hertzian theory has been employed most extensively for modeling. First significant theoretical work on contact dynamics was by Nayak [32], who developed some of

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Nomenclature

A_0	Nominal contact area
c	Damping coefficient
D	Fractal dimension
E	Elastic modulus of deformable surface
E'	Composite elastic modulus of equivalent rough surface
F	Amplitude of harmonic excitation
F_s	Restoring force of spring
\hat{F}	Normalized amplitude of harmonic excitation
f	Frequency index
f_{\max}	Maximum frequency index
G	Fractal roughness
g	Gravitational acceleration
k	Nonlinear stiffness of spring/deformable surface
L	Sample length of fractal surface
L_s	Cut off length of fractal surface
M	Number of superimposed ridges to construct the fractal surface
m	Mass of the block
n	Exponent value in the power law force-displacement relationship equation $P^* = k(\delta^*)^n$
P	Applied normal load on rough surface
P^*	Normalized applied normal load, $P^* = \frac{P}{A_0 E'}$
t	Time
u	Normalized vertical displacement
$u(\max)$	Maximum normalized vertical displacement (above which contact loss occurs)
z	Vertical displacement of block
z_s	Static vertical displacement of block due to its own weight
γ	Frequency density
δ	Displacement of contacting rough surface
δ^*	Normalized displacement of contacting rough surface, $\delta^* = \delta/L$
ν	Poisson's ratio
ζ	Damping ratio
τ	Normalized time, $\tau = t\omega_s$
ω	Harmonic forcing frequency
ω_s	Natural frequency of system at the static equilibrium position

the theoretical groundwork necessary for detailed physical explanations of experimentally observed phenomena in vibratory point contact by modeling a single-degree-of-freedom dynamic system and obtained analytical solutions by using a single term harmonic balancing method (HBM) [33]. Multi term HBM was also used by Ma et al. [34] while studying sphere-plane (Hertzian) contact model analytically as well as experimentally. Besides HBM, Method of Multiple Scales (MMS) [33] is used extensively by researchers for solving nonlinear differential equation of motion. Recently, Xiao et al. [35] used exact method, MMS and HBM to determine the natural frequency of undamped free vibration of a mass interacting with Hertzian contact stiffness. Hess and Soom [36,37] studied nonlinear vibrations at Hertzian contact as well as at the contact region formed between rough surfaces excited by the dynamic component of an externally applied normal load by MMS. Similar Hertzian contact problem for sub-harmonic and super-harmonic resonance of order two was investigated by Perret-Liaudet [38,39] using MMS. The condition for contact loss was taken into consideration for the first time in this work. Dynamic analysis of Hertzian contact using analytical and numerical methods was presented by number of researchers including Sabot et al. [40] and Perret-Liaudet

and Sabot [41]. Experimental analysis of sphere-plane contact with sinusoidal, random excitation and with subharmonic and superharmonic excitation was also carried out by some researchers [42–45]. However, very few research works is found on dynamic analysis of multi-asperity contact. Xiao et al. [46] recently modeled an elastic fractal surface contacting with a rigid flat surface, analyzed its force-displacement relation and studied its effect on the free and forced vibration responses of the system. Tian and Xie [47] investigated the dynamic contact stiffness at the interface between a vibrating rigid sphere and a semi-infinite transversely isotropic viscoelastic solid with an oscillating force superimposed onto a static compressive force.

In the present paper the effect of variation of material properties of the contacting fractal surfaces on free and forced vibration responses of the system is analyzed. A three-dimensional rough fractal surface is constructed using a modified two-variable Weierstrass-Mandelbrot function [15] and the force-deflection relationship of the rough surface contacting with a rigid flat surface is determined using the finite element analysis. The current study considers nominally smooth contact which are assumed to be frictionless, as the effect of friction arising out of asperity interactions and atomic friction are neglected. The equation of motion of the system is represented by the Helmholtz–Duffing equation using a third-order Taylor series expansion which is dependent on the power value of the force-normal displacement relationship. For different post-elastic material properties such as tangent modulus (assuming bilinear model) and yield strength values, the force displacement relationship is determined. The natural frequency of the undamped free vibration of the system is determined by numerical quadrature method and for the forced damped system, harmonic response amplitude is calculated numerically using Runge-Kutta method.

2. Fractal surface modeling

A realistic rough surface can be modeled by a 3D fractal surface topography [3]. In the present paper the fractal surface is generated with the help of modified two-variable Weierstrass-Mandelbrot function expressed in the following form

$$z(x, y) = L \left(\frac{G}{L} \right)^{D-2} \left(\frac{\ln \gamma}{M} \right)^{1/2} \sum_{\mu=0}^M \sum_{f=0}^{f_{\max}} \gamma^{f(D-3)} \times \left\{ \cos \phi - \cos \left[\frac{2\pi \gamma^f (x^2 + y^2)^{1/2}}{L} \times \cos \left(\tan^{-1} \left(\frac{y}{x} \right) - \frac{\pi \mu}{M} \right) + \phi \right] \right\} \quad (1)$$

where, L is the sample length, and G is a fractal height scaling parameter independent of frequency within the scale range. D, M and γ are fractal dimension, number of superimposed ridges to construct the surface and frequency density, respectively. For physical rough surfaces the value of fractal dimension is found to be between 2 and 3 [15]. Random phase angle is represented by ϕ . Frequency index is represented by f . Frequency index has a lower limit at zero for a truncated series of the height function and the upper limit is given by,

$$f_{\max} = \text{int} \left[\frac{\log(L/L_s)}{\log \gamma} \right] \quad (2)$$

Where, $\text{int}[\cdot]$ represents the maximum integer value of the number within the bracket and L_s is the cut off length. The values of sample length and cut-off length are set to be 1×10^{-6} m and 1.5×10^{-7} m respectively. The parameters M, γ and f_{\max} have values of 10, 1.5 and 5 respectively. Low values of G and high values of D signify smoother and highly dense surface profile and vice-versa. Using the above

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