Applied Surface Science 314 (2014) 610–621

Contents lists available at ScienceDirect

Applied Surface Science

journal homepage: www.elsevier.com/locate/apsusc

Box-counting methods to directly estimate the fractal dimension of a rock surface

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ARTICLE INFO

Article history:
Received 9 December 2013
Received in revised form 10 June 2014
Accepted 13 June 2014
Available online 11 July 2014

Keywords:
Rock surface
Box-counting dimension
Cubic covering
Takagi surface

ABSTRACT

Surfaces of rocks are usually not perfectly "smooth", and two box-counting methods, i.e. the conventional cubic covering method (CCM) and improved cubic covering method (ICCM), can directly describe the irregularities of a rock fracture surface without any approximate calculations. Our investigation showed that if the scale δ of covering cubes is greater than the sampling interval S_0, the CCM and ICCM cannot completely cover the object rough surface. Considering this, we presented two new cubic covering methods, namely the differential cubic covering method (DCCM) and relative differential cubic covering method (RDCCM) to directly evaluate the fractal dimension of a rough surface according to the definition of box-counting dimension. Experimentally, a 3D laser profilometer was used to measure the topography of a natural surface of sandstone. With the CCM, ICCM, DCCM and RDCCM, direct estimations of the fractal dimension of the rock surface were performed. It was found the DCCM and RDCCM usually need more cubes to cover the whole fracture surface than the CCM and ICCM do. However, the estimated fractal dimensions by the four methods were quite close. Hence, three Takagi surfaces with known fractal dimensions of 2.10, 2.50 and 2.90 were adopted to further examine the four box-counting algorithms. Results showed that for a low fractal dimension Takagi surface, the DCCM and RDCCM gave accurate results within the ranges determined by small covering scales, whereas the CCM and ICCM always overestimate the fractal dimension for all the potential scale ranges investigated in current work; for high fractal dimension surfaces, the CCM and ICCM provided very good results within the ranges determined by small covering scales, and oppositely, the DCCM and RDCCM cannot provide a good estimation of the fractal dimension within such scale ranges but can determine approximate results at large scales.

1. Introduction

The classic methods of solid mechanics assume the homeomorphism of deformation [1]. However, voids, cracks, faults, fractures and joints always exist in rocks as a result of various geological processes [2]. The mechanical properties of fractured rocks principally depend on the state of existing discontinuities [3], and moreover, the behaviors of these discontinuities are strongly affected by their surface characteristics [4]. Therefore, the quantitative description of the topography of a fracture surface is very important.

Material surfaces, especially the surfaces of rock-like materials, are usually not perfectly "smooth" and the irregularities in the form of valleys and convexities always exist [5]. Based on the fractal geometry [6], a theory to characterize the degree of irregularities, many researchers quantitatively described the morphology of rock fracture surfaces, and it has been confirmed that fracture surfaces in rocks exhibited a statistical fractal behavior in a certain scale range [7]. The fractal dimension D can be used to measure the irregularities and the degree of complexity of surface shape [5] and the intercept, in a log–log way, is an indicator of asperities [8]. In the early stage, in order to simplify the problem and avoid data acquisition difficulties, a linear sectional profile of a surface was widely characterized to grasp the roughness of a 3D surface [9]. The indirect measurement methods (slit island analysis, the divider and the self-affine variogram) were often employed to measure such sectional fracture profiles [10]. As a

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http://dx.doi.org/10.1016/j.apsusc.2014.06.152
0169-4332/

result, fractal dimensions measured by these methods fall in the range $1 < D < 2$ [10]. However, a one-dimensional analysis provides an incomplete and even biased characterization of a fracture surface [11]. Thus, the fractal dimension of a rough surface obtained by adding 1.0 to the fractal dimension obtained from a single sectional profile of that surface was widely adopted [11]. Such an approximation might be very close to the real fractal dimension of a fracture surface, but essentially, it is unacceptable [11]. In order to perform a direct measurement of the fractal dimensions of fracture surfaces, the measurement technique should be taken into account first [10]. As summarized in [12,13], the scanning electron microscopy (SEM) [14,15], atomic force microscopy (AFM) [16–19], mechanical stylus profilometry (MSP) [20] and non-contact laser profilometry (LPM) [21,22] are commonly used to conduct a topographical measure of a surface. Among these methods, the profilometric methods can provide quantitative topographical information of a rough surface [12,13]. Especially the non-contact optical instruments, such as a 3D laser profilometer, make it more convenient to obtain the morphology data of a rock fracture surface in a form of (x, y, z) coordinates. In the theoretical aspect, the triangular prism surface area method (TPSAM) [23], proposed in 1986, made it possible to conduct a direct measurement of the fractal dimensions of a fracture surface. In this method, a three-dimensional geometric equivalent of the "walking dividers" method in two dimensions was adopted [23]. Elevation values at the corners of cubes were employed to interpolate a center value. However, the elevation at the center of each grid cell is determined by linear interpolation of the four heights of the adjacent points. Thus, it is almost impossible to exactly calculate the true area of the fracture surface within the grid cell [11]. Considering this, the projective covering method (PCM) [10], which provides a similar fractal dimension result, was proposed. In this method, the real area surrounded by four points on the fracture surface is approximated by two triangles and every point for calculation of the approximate area can be assured to be on the fracture surface. Recently, Kwaśny [24] modified the PCM by introducing a more precise area calculation method. Nevertheless, the area of the fracture surface is also approximate. To accurately estimate the fractal dimension of a fracture, a new method, namely cubic covering method (CCM) [11] was presented according to the principle of the covering method. The most important consideration is that every point (which is essentially the sampling point) used is exactly located on the rough surface, thus the calculated fractal dimension is absolutely accurate. Based on the CCM, an improved cubic covering method (ICCM) was proposed [25]. The cubic covering procedure was implemented from a universal basis plane for each grid with a measurement scale δ for the ICCM.

Generally, for a box-counting method, it should be able to completely cover a fractal set [26,27]. However, it is almost impossible to ensure that the maximum (minimum) one among the height values of the four intersection points is exactly the maximum (minimum) height of the irregular surface area within the scale δ when δ is larger than the sampling interval of the laser scanning due to the complexity of surface shape. Therefore, both the CCM and ICCM, strictly speaking, cannot totally cover a fractal set. Considering this, we presented two new cubic covering methods, namely the differential cubic covering method (DCCM) and relative differential cubic covering method (RDCCM). Both the DCCM and RDCCM can totally cover the fracture surface. Later, a laser profilometer was then used to obtain the data set of a fracture surface of sandstone. With the data set, we directly determined the fractal dimension of the fracture surface of sandstone by the CCM, ICCM, DCCM and RDCCM, respectively, and quite similar results were obtained. Finally, series of rough surfaces which have known fractal dimensions were generated based on the Takagi function to validate the applicability of the four covering methods.

2. Box-counting algorithms

2.1. Box-counting dimension

Due to the ease of mathematical calculation and empirical estimation, box-counting dimension is one of the most widely used dimension [27]. To find the box-counting dimension of F, a non-empty bounded subset of R^n, one may draw a mesh of squares or boxes of side δ and count the number $N(\delta)$ that overlap the set [27]. Through changing the scale δ, different values of $N(\delta)$ can be obtained. The total number $N(\delta)$ of cubes depends on the used measurement scale, δ. If the fracture surface exhibits the fractal behavior, the relation between $N(\delta)$ and δ is given by

$$N(\delta) \sim \delta^{-D}, \tag{1}$$

where D is the fractal dimension of the object fracture surface. Log transformation of this simple power law yields a straight line with slope $-D$. Therefore, the box-counting dimension can be estimated by the gradient of the graph of $\ln N(\delta)$ against $\ln \delta$ given by

$$D_B = -\lim_{\delta \to 0} \frac{\ln N_\delta(F)}{\ln \delta}. \tag{2}$$

The number $N(\delta)$ of δ-mesh cubes that intersect a set is an indicator how spread out or irregular the set is when examined at a scale of δ [27]. The box-counting dimension shows how rapidly the irregularities develop as δ tends to 0 [27].

2.2. The conventional cubic covering method and its improved version

The conventional cubic covering method [11] provides a very simple way to directly calculate the fractal dimension of a rough surface. It is assumed that there exists a regular cube grid on the plane XOY (see Fig. 1a), and that in each grid cell with scale δ, four intersection points correspond to four heights of a fracture surface: $h_1(i,j)$, $h_2(i,j+1)$, $h_3(i+1,j)$, and $h_4(i+1,j+1)$ (where $1 \le i, j \le n-1$, n is the total number of sampling points on each individual profile on a fracture surface). If the cubes with the measurement scale δ are used to cover the irregular surface area within the scale δ, the maximum difference among $h_1(i,j)$, $h_2(i,j+1)$, $h_3(i+1,j)$, and $h_4(i+1,j+1)$ will determine the number $N_{i,j}$ of the required cubes:

$$\begin{aligned} N_{i,j} = \mathrm{INT}\{\delta^{-1}[&\max(h_1(i,j), h_2(i,j+1), h_3(i+1,j), h_4(i+1,j+1)) \\ &- \min(h_1(i,j), h_2(i,j+1), h_3(i+1,j), h_4(i+1,j+1))] + 1\}, \end{aligned} \tag{3}$$

where INT rounds the element to the nearest integer towards positive infinity. Then, the total number of cubes required for covering the whole fracture surface is

$$N(\delta) = \sum_{i,j=1}^{n-1} N_{i,j}. \tag{4}$$

Apparently, the covering process in each grid cell is always implemented from the lowest point among the four intersection points. Zhang et al. [25] pointed out that it is difficult to well describe the complexity of a rough surface with such a covering procedure, and therefore, they proposed an improved cubic covering method (ICCM, the schematic view was shown in Fig. 1b). The ICCM gives the number $N_{i,j}$ of cubes need to cover the irregular surface area within the scale δ by