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Image restoration with l_2-type edge-continuous overlapping group sparsity ☆

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ABSTRACT

It is important and necessary to take account of the non-zero pattern in image sparsity representation. In this paper, we present an image restoration model by introducing a novel edge-continuous overlapping group sparsity regularizer (EC-OGS), based on our observation that the non-zero entries in an image gradient domain often distribute along its edges. The model is solved by the ADMM (alternating direction method of multipliers), where a fast novel algorithm is proposed for computing the proximal operator in solving the subproblem with EC-OGS regularizer. The proposed model can be applied to various image restoration tasks including denoising, deblurring, and edge-detecting. The numerical experiments demonstrate the effectiveness of our method in terms of PSNR, visual effect and edge preserving.

1. Introduction

Image restoration is an important task in the field of image processing and usually formulated as a linear inverse problem. The objective of image restoration is to estimate an image u from an observed image $f \in R^d$ by the following minimization problem:

$$\min_u \frac{1}{2}\|Hu - f\|_2^2 + \lambda\phi(u) \tag{1}$$

where $\phi(u)$ is a regularization term, H is a linear operator which is typically an identity operator in image denoising, a projection operator in image inpainting or a blurring operator in image deblurring. The most two important subjects of this model are about how to design a regularizer $\phi(u)$ effectively and how to find good computation methods for the model. The regularizer is usually with some kinds of assumptions, such as the solution having the feature of sparsity or group sparsity. In the field of sparse representation, l_1 norm usually acts as the relaxed convex regularizer for l_0 which is exactly the norm for sparse coding. In recent years, sparsity-based approaches have led to promising results for various image restoration problems. The sparsity-based regularization problems can be classified into the following two kinds: the first one assumes that the unknown image u has the nature of sparse representation and can be synthesized by a few atoms in a given dictionary ϕ(synthesis-based sparsity problems), while the second one assumes that the analysis coefficients Du(D is the analysis operator) in the analysis domains are sparse (analysis-based sparsity problems). They can be modeled as $\min_u \frac{1}{2}\|H\phi\alpha - f\|_2^2 + \lambda\|\alpha\|_1$ and $\min_u \frac{1}{2}\|Hu - f\|_2^2 + \lambda\|Du\|_1$ respectively. Elad and Aharon [17] proposed K-SVD algorithm for learning a dictionary for image sparse representations. Nonlocal-patches based sparse representation approaches [1,25,29,30,43] have made a great success in the field of image restoration. [1,29,30] proposed BM3D algorithm for image denoising based on an enhanced sparse representation in transform domain. Dong and Zhang [43] proposed a nonlocally centralized sparse representation (NCSR) model to improve the performance of sparse representation-based image restoration by suppressing the sparse coding noise. Mairal et al., [25] present simultaneous sparse coding as a framework by two approaches, namely exploiting self-similarities and learning dictionary. Dong et al., [42] incorporate the image nonlocal self-similarity into sparse representation for image interpolation. These methods are mostly classified into the synthesis-based sparsity problems. In this paper, we pay more attention to the analysis-based sparsity problems in image gradient domain. We give a review about analysis-based approaches as following.

A definition of $\phi(u)$ as a l_2-type norm named Tikhonov regularizer ($\phi(u) = \|\nabla u\|_2, \nabla$ is the gradient operator) was proposed by Tikhonov [40]. Although it has the virtue of simple computation, the regularizer overly smoothes the edges which are very important features in the natural images. To overcome smearing edges, a regularizer based on total variation (TV) was proposed in [38] (in which $\phi(u) = \sqrt{\nabla u_x^2 + \nabla u_y^2}$, the regularizer often called the isotropical TV regularizer). The well-known model by this regularizer is called the ROF model. Esedoglu and Osher [18] also proposed the anisotropical ROF model where $\phi(u) = \|\nabla u_x\|_1 + \|\nabla u_y\|_1$. A remarkable advantage of TV regularizer is good edge-protecting. Due to this, it is

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widely used in different applications of image processing, such as blind deconvolution, inpainting and superresolution; see [13] for an overview. Many algorithms have been proposed for solving the ROF model. A semi-implicit gradient descent algorithm [8] is proposed by studying a dual formulation of TV denoising problem. Goldstein and Osher [21] also proposed a fast and stable method named the split Bregman algorithm. Introducing an auxiliary variable to replace f, another Fast TV method is proposed in [24]. In [6], a fast iterative shrinkage–thresholding algorithm (FISTA) for the ROF problem is studied. Afonso et al. [3] proposed an augmented Lagrangian shrinkage algorithm (SALSA) according to the alternating direction method of multipliers (ADMM). More recently, an effective method [9] is proposed by imposing a step of box constraints on the ROF model.

Although the TV regularizer has been proven to be extremely effective in various image processing applications, it often develops false edges which resulting in staircase artifacts [11]. Besides, as the numerical example (Fig. 2) in [12], the TV regularizer prefers round corners. Hence, TV-based methods do not preserve the structure, details, corners and textures of an image well. To overcome this shortcoming, high-order total variation regularizers like second-order [31] and fourth-order [35] partial differential operators were considered to avoid merely piecewise constant regions. To protect edges and fine details, Chao and Tsai [14] proposed a diffusion model incorporates local gradient and gray-level variance. Besides, a nonlocal TV regularizer [46] and a reweighted nonlocal TV regularizer [41] both based on patch-similarity was also proposed to avoid staircase artifacts. Actually, other different types of regularizers, such as the frame-based l_1 regularizer [7] and the Mumford–Shad regularizer [37] have been proven useful to image processing. As a kind of l_2-based regularizer, Mumford–Shad regularizer ($\phi(u) = \|\nabla u\|_2^2$) forces the edges to be smooth but does not force spatial coherence such as edge direction compatibility or edge connectivity. Unlike the TV regularizer, frame-based l_1 regularizer [7] is considered in a tight frame domain instead of gradient domain. Investigating the reason for the staircase artifacts and round corners by the ROF model from the view of sparsity, it is probable that the l_1 norm in the ROF model does not embody the relationship between those non-zero entries of the spare solution well. In other words, those non-zero entries are almost independent of each other. There are some examples for showing the shortcomings of l_1 norm. In the field of face recognition, robustness of occlusions will be improved by considering as features or sets of pixels that form small regions in face images [28]. However, the l_1-norm regularizer fails to encode this special constraint. Extracting bounding boxes is necessary in the task of object and scene recognition and these boxes should to be extracted by respecting the distribution of the pixels in the images, therefore, an unstructured regularizer like l_1-norm may not be suitable. Similarly, in neuroimaging area, the fMRI (functional magnetic resonance imaging) interests in localizing areas and should construct the brain in three-dimensional space [2,22,44], so the voxels should have a special localized spatial organization. But the optimization problem regularized by the l_1-norm will ignore the spatial configuration and will not obtain good results.

To further improve the solutions and avoid the shortcomings discussed above, more recent studies suggested to take the underlying structure of solutions into account while using sparsity coding methods [16,23,26]. A series of solutions with special group sparsity structure have been proposed in [16,23,26,36]. Recently, OGS (overlapping group sparsity) has been considered for image or signal processing, see [4,5,15,16,19,27,32–34,39]. Liu et al. [34] set $\phi(u)$ in (1) to be the OGS-TV (overlapping group sparsity total variation) and proposed an image restoration model, which was resolved by ADMM framework which includes majorization-minimization (MM, [20]) as an important step. However, the algorithm is slow because the MM method needs an inner iteration. Liu et al. [33] proposed a new explicit thresholding/shrinkage formula for one class of regularization problem with the OGS-TV. Because the OGS-TV is defined as the sum of l_2-norms of all square $K*K$-point groups in the gradient domain, the solutions obtained by the OGS-TV unfortunately have no any specific distribution of non-zero entries.

Based on the analysis for the l_1-norm regularizer and the OGS-TV regularizer, in this work we focus on modeling the distribution of the non-zero entries in image gradient domain. It is known that edges are always continuous curves on which the gradients are non-zeroes. Based on this observation, we firstly design special atomic vectors, each with a support set of k continuous points along a straight line in gradient domain. Then a new regularizer minimizing the sum of l_2-norms of these atomic vectors is proposed for image restoration problems. An image restoration model by this new regularizer is presented and solved by ADMM, which can produce better restored images than relevant methods. The main contributions of this paper are as follows: First, a novel regularizer named EC-OGS (Edge-continuous overlapping group sparsity) is proposed for image processing; Second, we propose a novel algorithm for computing the proximal operator of the optimization subproblem with the new regularizer EC-OGS.

The rest of this paper is organized as follows. In Section 2, we propose our edge-continuous overlapping group sparsity regularizer in the gradient domain and the relevant image restoration model. In Section 3, under the ADMM frame, we proposed an optimization algorithm for this model. A fast novel method for computing the proximal operator of the subproblem with EC-OGS is proposed in Section 4. In Section 5, to demonstrate the effectiveness and advantage of our method over existing methods, we give a number of numerical experiments on various image restoration tasks including image denoising, deblurring, and edge-detecting. Finally, we conclude with a discussion on the proposed regularizer and point out some future research directions.

2. Edge-continuous overlapping group sparsity regularizer and image restoration model

As we all know, edges of an image are commonly some continuous curves, on which the gradient magnitudes are generally non-zeros in the image gradient domain. Based on this observation, atomic vectors with the support set of k continuous points at most along a straight line in gradient domain are designed in this paper. We proposed a regularizer in gradient domains by minimizing the sum of the l_2 norms of these atomic vectors. For an $M*N$ image, we partition its gradient into $4kd$ vectors, where $d = MN$. In particular, let $z \in R^d$ be a gradient, and we decompose it into

$$z = \frac{1}{k}\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{p=1}^{4k} z^p(i,j)$$

where, $z^p(i,j) \in R^d$ is a sparse vector satisfying $\|z^p(i,j)\|_0 \le k$ and its support forms a straight line containing (i,j) along directions of angle multiples of $\pi/4$. Fig. 1(a) and (b) shows the $4k$ overlapped support sets on point (i,j) when $k=3$ and $k=2$ respectively, named $G_1, G_2, \ldots, G_{4k}$. In Fig. 1(a), there are 12 support sets. Each along one direction and includes three continuous points. For example, G_1 includes $\{(i,j-1),(i,j),(i,j+1)\}$ along horizontal direction and G_6 includes $\{(i,j),(i+1,j-1),(i+2,j-2)\}$ along diagonal direction from top-right to bottom-left. We denote the relevant vectors corresponding to those support sets $G_1, G_2, \ldots, G_{4k}, k=3 \quad or \quad k=2$ in Fig. 1 as $z^p(i,j), p = 1,2,\ldots,4k$.

Considering the ∇u_x or ∇u_y of an image as a vector concatenated by its all columns, the pattern of each vector $z^p(i,j) \in R^d$, $p = 1,2,\ldots,4k$ is organized as the following formula:

$$z^p(i,j) = \overbrace{(0,\ldots,z^{p_1},0,\ldots,z^{p_2},0,\ldots,z^{p_k},\ldots)}^{d=MN} \quad (2)$$

where, $support(z^p(i,j)) = (p_1,p_2,\ldots,p_k) = indexes(G_p)$. That means the non-zero indexes $p_1,p_2,\ldots,p_k$ of the vector $z^p(i,j)$ is