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3D rotation invariants of Gaussian–Hermite moments ☆

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ABSTRACT

3D rotation invariants based on orthogonal Gaussian–Hermite moments are proposed in this paper. We present an elegant and easy theoretical derivation of them. At the same time we prove by experiments that the Gaussian–Hermite invariants have better numerical stability than the traditional invariants composed of geometric moments.

1. Introduction

Pattern classification and object recognition play vital roles in image processing and computer vision. Generally, recognition is achieved by seeking descriptors that can represent the object regardless of certain transformations and/or deformations. Moment invariants were proved to be very powerful tools for feature representation and it has been demonstrated many times that moment invariants perform effectively in object recognition [1].

So far, various kinds of moment invariants to spatial transformations of the object have been proposed. Among all transformations that have been studied in this context, *rotation* plays a central role. Being a part of rigid-body transformation, object rotation is present almost in all applications, even if the imaging system is well set up and the experiment has been prepared in a laboratory. On the other hand, rotation is not trivial to handle mathematically, unlike for instance translation and scaling. For these two reasons, invariants to rotation have been in focus of researchers since the beginning.

With the rapid progress of applied mathematics, computer science and sensor technology, 3D imaging comes into engineering and practice due to its more flexible and precise descriptions of 3D objects. Undoubtedly, developing rotation invariants for 3D images has become a hot topic in the computer vision community. However, 3D rotation is more difficult to handle than its 2D counterpart, since it has three independent parameters. That is probably why only few papers on 3D rotation moment invariants have appeared so far. The first attempts to derive 3D rotation moment invariants are relatively old. Sadjadi and Hall [2] explored ternary quadratics extensively and derived three translation, rotation and scaling (TRS) moment invariants. Guo [3] proved the results of Sadjadi and Hall in the different way and he derived more invariants to translation and rotation in 3D space. Cyganski and Orr [4] applied tensor theory to derive 3D rotation invariants. This method was also mentioned by Reiss [5], who used invariant image features to recognize planar objects. Xu and Li [6] developed the invariants in both 2D and 3D space based on geometric primitives, such as distance, area, and volume. Galvez and Canton [7] employed normalization approach. The object is transformed into the coordinates given by eigenvectors of the second-order moment matrix and its transformed moments are taken as invariants. A modification of this method appeared in [8], where a slightly different moment matrix is used for normalization. Another method to derive 3D rotation invariants is based on complex moments [9,10]. Recently, Suk and Flusser [11] proposed an automatic algorithm to generate 3D rotation invariants from geometric moments up to an arbitrary order.

Although moments are probably the most popular 3D shape descriptors, it should be mentioned that they are not the only features providing rotation invariance. For example, Kakarala and Mao [12] used the bispectrum well-known from statistics for feature computation. Kazhdan [13] used an analogy of phase correlation based on spherical harmonics for comparison of two objects. In this particular case it was used for registration, but can be also utilized for recognition. In [14], the authors used amplitude coefficients as the features. Fehr [15] used the power spectrum and bispectrum computed from a tensor function describing an object composed of patches. In [16], the same author employed local binary patterns and in [17] he used local spherical histograms of oriented gradients.

In comparison with traditional geometric or complex moments, the outstanding advantage of orthogonal moments is their better numerical stability, limited range of values, and existing recurrent relations for their calculation. Hence, several authors have tried to derive the 2D invariants from orthogonal moments. In 3D, however, the

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situation is more difficult than in 2D, but one can still expect that 3D orthogonal moments preserve their favorable numerical properties. There exist polynomials orthogonal inside a unit ball and others that are orthogonal on a unit cube. Seemingly, the polynomials defined on a unit ball are more convenient for deriving rotation invariants because the ball is mapped onto itself and the polynomials are transformed relatively easily under rotation. This approach was used by Canterakis [18] who employed 3D Zernike moments.

In this paper, we propose rotation 3D invariants from Gaussian–Hermite moments. To derive them, we used an approach that we already successfully applied in 2D [19]. We prove that the transformation of Gaussian–Hermite moments under rotation can be deduced indirectly, without explicit investigation of this transformation. Under our knowledge, Gaussian–Hermite polynomials are the only ones offering this possibility. Hence, we prove in the paper that the rotation invariants from Gaussian–Hermite moments have the same forms as those of rotation invariants from geometric moments in 3D space. This is an important conclusion because it allows us to reduce rotation invariant derivation from Gaussian–Hermite moments to that from geometric moments in 3D space, which are much easier to develop but we still benefit from the numerical stability of Gaussian–Hermite moments.

The core idea of the paper and its main theoretical achievement expressed by Theorem 1 is similar to that presented in [19] for a 2D case. It should be, however, stressed that the transition from 2D to 3D is not generally straightforward and easy. The rotation in 3D has three degrees of freedom comparing to a single parameter of a 2D rotation. Hence, any 3D mathematical objects and structures somehow related to rotation are far more rich than in 2D. Another difference that also makes the 3D problem more complicated is that rotation in 3D is not commutative. These are the reasons why the generalization from 2D to 3D cannot be done automatically but should always be carefully studied. Such studies sometimes discover an analogy with 2D (which is the case of this paper) and sometimes end up with different results.

The rest of the paper is organized as follows. Section 2 gives a general introduction to 3D rotation. The latest achievement about rotation invariants from geometric moments in 3D space is also recalled in this section. Section 3 reviews Gaussian–Hermite moments and gives two theorems according to which we can use the formations of geometric invariants to build rotation invariants of Gaussian–Hermite moments. Numerical experiments are presented in Section 4. Finally, Section 5 concludes the paper.

2. 3D rotation and its invariants

To describe a rotation in 3D space, we use extrinsic Tait–Bryan angle convention $(z-y-x)$ [20]. We consider the rotation along z axis by angle α, along y axis by angle $-\beta$, and along x axis by angle γ. Hence, a general 3D rotation can be directly represented by a matrix multiplication

$$\mathbf{R} = \mathbf{R_x}(\gamma)\mathbf{R_y}(-\beta)\mathbf{R_z}(\alpha). \tag{1}$$

Any rotation in 3D space can be decomposed into three successive rotations as defined by Eq. (1). Thanks to this, it is sufficient to consider elementary rotations along the axes only.

In 3D space, geometric central moment of order $(p+q+r)$ is defined

$$\mu_{pqr} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-x_c)^p(y-y_c)^q(z-z_c)^r f(x,y,z)\,dxdydz, \tag{2}$$

where the centroid of the image $f(x,y,z)$ is calculated by $x_c = m_{100}/m_{000}$, $y_c = m_{010}/m_{000}$, and $z_c = m_{001}/m_{000}$. Recently, Suk and Flusser [11] proposed and implemented an automatic method for generating 3D rotation invariants from geometric moments. Their complete results are summarized in [21]. A list of 1185 irreducible rotation invariants in 3D space is available there. These invariants are built up from the moments of order 2 up to order 16. 3D rotation invariants of geometric moments are potential tools for the applications, such as object recognition and image retrieval. However, poor numerical stability exposes when the order of the invariant increases to a certain number. Hence, it is necessary to develop 3D rotation invariants based on orthogonal moments, which generally have better numerical stability than geometric moments.

3. 3D rotation invariants from Gaussian–Hermite moments

3.1. Gaussian–Hermite moments

The pth degree Hermite polynomial is defined by

$$H_p(x) = (-1)^p \exp(x^2)\frac{d^p}{dx^p}\exp(-x^2). \tag{3}$$

Hermite polynomials can be efficiently computed by the following 3-term recurrence relation:

$$H_{p+1}(x) = 2xH_p(x) - 2pH_{p-1}(x) \quad \text{for } p \geq 1, \tag{4}$$

with the initial conditions $H_0(x) = 1$ and $H_1(x) = 2x$. Hermite polynomials are orthogonal on $(-\infty, \infty)$ with a Gaussian weight function

$$\int_{-\infty}^{\infty} H_p(x)H_q(x)\exp(-x^2)dx = 2^p p!\sqrt{\pi}\,\delta_{pq}, \tag{5}$$

where δ_{pq} is the Kronecker delta. A weighted and normalized version, which is actually a scaled Hermite function, is usually used in practice

$$\widetilde{H}_p(x;\sigma) = (2^p p!\sqrt{\pi}\sigma)^{-1/2}H_p(x/\sigma)\exp(-x^2/2\sigma^2). \tag{6}$$

Gaussian–Hermite moment is defined with (6) being its basis function. The system (6) is not only orthogonal but also orthonormal, so it is convenient to conduct image reconstruction from the corresponding moments. However, when we multiply $\widetilde{H}_p(x;\sigma)$ in x direction and $\widetilde{H}_q(y;\sigma)$ in y direction, the product depends not only on the sum $p+q$, but also on the product $p!q!$; therefore, we must remove it from the basis function

$$\widehat{H}_p(x;\sigma) = H_p(x/\sigma)\exp(-x^2/2\sigma^2). \tag{7}$$

Fig. 1 shows such non-coefficient basis functions (7) of order 8 with different σ. We call the moments with respect to the basis functions (7) non-coefficient Gaussian–Hermite moments. For an image $f(x,y,z)$ in 3D space, its non-coefficient Gaussian–Hermite moment of order $p+q+r$ is defined as

$$\eta_{pqr} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\widehat{H}_p(x;\sigma)\widehat{H}_q(y;\sigma)\widehat{H}_r(z;\sigma)f(x,y,z)dxdydz. \tag{8}$$

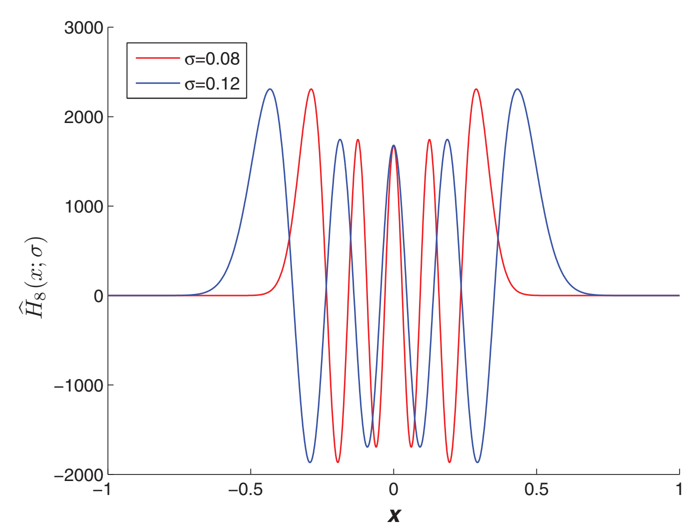

Fig. 1. Non-coefficient basis functions of the 8th order with different σ.