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Asymmetric power distribution model of wavelet subbands for texture classification ☆

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ABSTRACT

The generalized Gaussian distribution (GGD) is a well established statistical model for wavelet subband characterization used in several applications. However, it is not really suitable for eventual asymmetry of probability density functions. Therefore, in this paper we propose to exploit the asymmetric power distribution (APD) which is a more general and flexible model than the GGD. The APD parameters are estimated through the maximum-likelihood estimation. A supervised texture classification problem is proposed as an application in this work. It is based on the Bayesian framework which has led to the definition of the closed form of the corresponding Kullback–Leibler divergence considered as a similarity measure. To validate the APD model, the goodness-of-fit using the classical Kolmogorov–Smirnov test is used. Finally, classification results on four databases demonstrate the interest of the proposed approach.

1. Introduction

Over the last decade, numerous works for texture analysis have been proposed using multi-scale/multi-orientation image representations [1–8]. The use of these representations is justified, on the one hand, by the numerous studies showing that decompositions such as wavelet transforms are similar to the mechanism of human visual perception [9–11]. On the other hand, there is a textural diversity in natural images in terms of the orientation as well as the dependence of the texture appearance with respect to the observation scale. Thereby, wavelet transforms are an effective tool to characterize texture information as they capture details at different scales and orientations.

Several descriptors based on wavelet transforms have been proposed like co-occurrence matrices [12] and the energy and standard deviation of each wavelet subband [13]. Other methods rely on local descriptors like multifractal analysis [14–17]. The wavelet-based multifractal analysis leads to good performances for texture classification with robustness to rotation and scale changes. The probabilistic approach, used by many researchers [2,3,8,18,19], consists in providing a parametric probability density function (PDF) which fits the observed data and using the Kullback–Leibler divergence (KLD) as the measure of similarity between PDFs. This approach is well-founded in terms of classification. Indeed, the probabilistic framework has proven to be asymptotically equivalent to the optimal Bayesian classification [3,20]. Hence, the aim of this work is the study of the accurate probabilistic modeling of wavelet subbands with an application for texture classification.

A conventional scheme of texture analysis consists in representing each wavelet subband by a parametric probabilistic model that represents the marginal statistics of subbands. Mallat has introduced the use of the generalized Gaussian distribution (GGD) to model wavelet coefficients in his pioneering work introducing the multiresolution decomposition as a design method of discrete wavelet transforms [21]. Since then, several studies have considered the GGD as *a priori* knowledge of wavelet subband statistics, leading to good results for example in video coding [22], in image denoising [23], in change detection [24], in face recognition [25] and in texture retrieval and classification [2,3,8,26]. The GGD is widely used because of the almost heavy tailed aspect of the empirical probability density functions (PDF) of subbands. Since the marginal statistics of wavelet coefficients are highly non-Gaussian, the GGD is adequate to represent either the leptokurtic or platykurtic behavior of the marginal distributions of subbands.

However, the GGD presents limitations for modeling the eventual asymmetry of subband PDFs as it is symmetric by definition. Experiments have revealed that the assumption of symmetry of subband PDFs is often violated [4,18]. In order to take into account the asymmetry of subband PDFs we propose to use the asymmetric power distribution (APD), the GGD of which is a particular case. To highlight the interest in using the APD, we propose to investigate its goodness-of-fit to the observed statistics of the wavelet subbands. In addition,

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we use the Kullback–Leibler divergence as a similarity measure to adopt the APD model in a Bayesian framework for texture classification.

The sequel of the paper is organized as follows. In Section 2 the asymmetry of subband empirical distributions is described and the APD model is introduced. In Section 3 a reminder of the Bayesian framework is given and the associated Kullback–Leibler divergence used as the similarity measure is detailed. Then, the classifiers under consideration are presented. Section 4 provides the experimental results. Finally, Section 5 gives some concluding remarks and the outlook on future works.

2. Modeling asymmetric wavelet subband distributions

The modeling of marginal wavelet subband probability density functions for texture analysis is a classical approach. The usual model considered is the generalized Gaussian distribution (GGD), although authors have used the symmetric α-stable distribution [19,27] or Bessel K forms [28] to improve subband characterization. However, these models are by definition symmetric with heavy tails (leptokurtic) while empirical PDFs can be asymmetric [4,18] and even platykurtic although they are often leptokurtic. Thus, we propose the use of the asymmetric power distribution (APD) model which confers more flexibility to fit different platykurtic or leptokurtic, symmetric or asymmetric empirical PDFs.

2.1. Highlighting asymmetry

To show the eventual asymmetry of a subband PDF, we provide an illustrative example of this tail behavior. Fig. 1 shows an example of texture and Fig. 2 its wavelet transform using Daubechies filter 'db2' at the first scale decomposition. A_1, H_1, D_1, V_1 stand respectively for the approximation, the horizontal details, the diagonal details and the vertical details at scale 1.

From Fig. 3, it is obvious that the estimated GGD fails to fit the tails of the empirical PDF of the detail subbands which are asymmetric whereas the APD (presented in Section 2.2) fits them better.

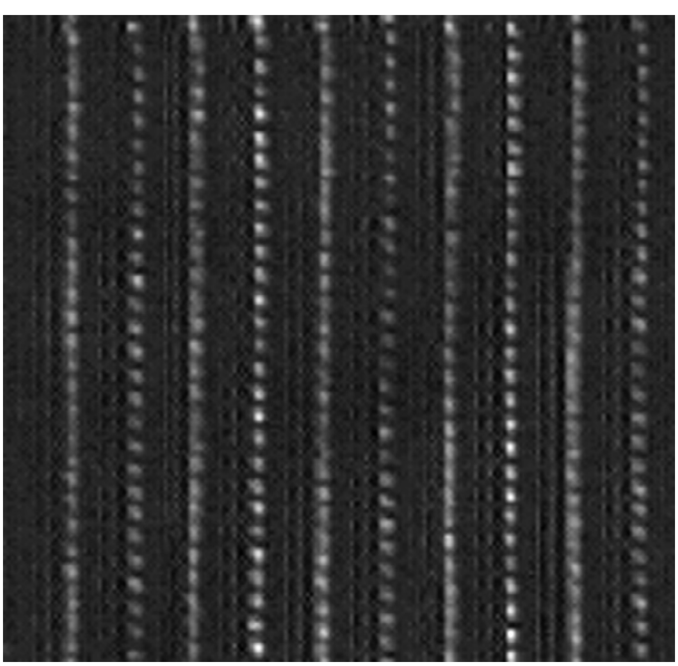

Fig. 1. Example of canvas texture.

To provide a rigorous quantitative illustration of subband PDF asymmetry we propose to consider the skewness which is commonly used to measure this behavior. Fig. 4 represents the distribution of the skewness of 5760 subbands resulting from three scale wavelet decomposition of the MIT VisTex texture database [29]. Although 44% of subbands have a skewness close to zero, several others (56%) have a significant asymmetry coefficient.

As a last way to illustrate the asymmetry of the subband PDFs, we consider the (skewness,kurtosis) scatterplot of the aforementioned 5760 wavelet subbands. The skewness/kurtosis plan in Fig. 5 shows how the APD is more suitable than the GGD to fit the observed (skewness,kurtosis) couple. We should not forget that the skewness is the third standardized moment while the kurtosis is the fourth. Hence, the skewness/kurtosis plan is a kind of higher-order statistic (HOS) fitting. In this plan, the GGD is represented by the blue dash-dot line while the APD covers all the space above the black dashed line which represents the kurtosis limit corresponding to that of a uniform PDF. It can be deduced from this HOS fitting that the APD tracks the observed statistics better than the GGD.

2.2. Asymmetric power distribution

2.2.1. Definition

Following [30], we propose to define the PDF of the asymmetric power distribution (APD) as

$$f_{\mathrm{APD}}(x) = \begin{cases} \dfrac{\beta}{\alpha(\gamma+1)\Gamma(1/\beta)} \exp\left[-\left(\dfrac{|x|}{\alpha\gamma}\right)^{\beta}\right], & x < 0 \\[2ex] \dfrac{\beta}{\alpha(\gamma+1)\Gamma(1/\beta)} \exp\left[-\left(\dfrac{|x|}{\alpha}\right)^{\beta}\right], & x \geq 0 \end{cases} \quad (1)$$

where γ is the asymmetry ratio, α the scale parameter and β the shape parameter ; $\Gamma(z)$ is the usual gamma function.

The APD expands the GGD when $\gamma = 1$.

In order to estimate the APD parameters (γ, α, β) we propose to use the maximum-likelihood estimation (MLE).

2.2.2. Parameter estimation

Under the hypothesis that the L-sized wavelet subband data $\mathbf{w} = (w_1, \ldots, w_L)$ are an independent and identically distributed sequence from an APD model with parameters (γ, α, β) to be estimated, the log likelihood function of the sample $\mathbf{w}$ is:

$$\mathrm{ll}(w_1, \ldots, w_L) = L \ln \frac{\beta}{\alpha(\gamma+1)\Gamma(1/\beta)} + \sum_{\substack{i=1 \\ w_i<0}}^{L} \left(\frac{|w_i|}{\alpha\gamma}\right)^{\beta} + \sum_{\substack{i=1 \\ w_i>0}}^{L} \left(\frac{|w_i|}{\alpha}\right)^{\beta}, \quad (2)$$

Setting its derivatives with respect to γ, α and β to zero we obtain:

$$\hat{\gamma} = \left(\frac{\sum_{\substack{i=1 \\ w_i<0}}^{L} |w_i|^{\hat{\beta}}}{\sum_{\substack{i=1 \\ w_i>0}}^{L} |w_i|^{\hat{\beta}}}\right)^{\frac{1}{\hat{\beta}+1}}, \quad (3)$$

$$\hat{\alpha} = \left[\frac{\hat{\beta}}{N}(1+\hat{\gamma}) \sum_{\substack{i=1 \\ w_i>0}}^{L} |w_i|^{\hat{\beta}}\right]^{\frac{1}{\hat{\beta}}}, \quad (4)$$