Pattern Recognition Letters 32 (2011) 910–918

Contents lists available at ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

The thin plate spline robust point matching (TPS-RPM) algorithm: A revisit

Jinzhong Yang *

Department of Radiation Physics, The University of Texas MD Anderson Cancer Center, Houston, TX 77030, USA

ARTICLE INFO

Article history:
Received 17 September 2009
Available online 1 February 2011
Communicated by M. Kamel

Keywords:
Robust point matching (RPM)
Thin plate splines (TPS)
Feature extraction
Outliers
Non-rigid registration

ABSTRACT

This paper reviews the TPS-RPM algorithm (Chui and Rangarajan, 2003) for robustly registering two sets of points and demonstrates from a theoretical point of view its inherent limited performance when outliers are present in both point sets simultaneously. A double-sided outlier handling approach is proposed to overcome this limitation with a rigorous mathematical proof as the underlying theoretical support. This double-sided outlier handling approach is proved to be equivalent to the original formulation of the point matching problem. For a practical application, we also extend the TPS-RPM algorithms to non-rigid image registration by registering two sets of sparse features extracted from images. The intensity information of the extracted features are incorporated into feature matching in order to reduce the impact from outliers. Our experiments demonstrate the double-sided outlier handling approach and the efficiency of intensity information in assisting outlier detection.

1. Introduction

Surface matching or image registration problems frequently arise in the medical imaging and computer vision domains (Zitová and Flusser, 2003; Maintz and Viergever, 1998). The registration is a process to establish point correspondence between two surfaces or images. Both surface matching and image registration can boil down to searching for an optimal transformation as well as the correspondence between two sets of points. Typically, in the image registration case, the points could be pixel locations of the images, or some sparse features extracted from the images. In general, the resulting point matching problem is difficult since both the correspondence and transformation are unknown. Furthermore, various factors such as the noise, outliers, and deformations can make the problem even more difficult. In literature, researchers approached this problem normally based on an iterative estimation framework, since optimal solution of the correspondence (transformation) is much easier to produce if the transformation (correspondence) is known. The well-known iterated closest point (ICP) algorithm (Besl and McKay, 1992) is one example to solve for an underlying rigid transform between two point sets. Later developed methods extended the fundamental idea of ICP to address non-rigid mappings of two point sets (Cross and Hancock, 1998; Rohr et al., 2001; Belongie et al., 2002; Chui and Rangarajan, 2003; Zheng and Doermann, 2006; Taron et al., 2009; Gerogiannis et al., 2009; Boughorbel et al., 2010). Some of these methods even possess the capability to detect outliers automatically (Chui and Rangarajan, 2003; Zheng and Doermann, 2006). The outliers here are specifically referred to those points in one point set that have no correspondence in the other point set. These outliers should be identified and rejected during the matching process. In this paper, we particularly focus on solving the point matching problem when a considerable amount of outliers exist simultaneously in both point sets.

The TPS-RPM algorithm (Chui and Rangarajan, 2003) was originally developed to solve the point matching problem in the presence of outliers. This method takes advantage of the softassign technique (Rangarajan et al., 1997) and the deterministic annealing technique (Yuille and Kosowsky, 1994) for robust point matching and it is able to handle outliers existing in the fixed point set (the point set is fixed during matching). However, the inherent structure of TPS-RPM algorithm does not efficiently handle outliers that simultaneously exist in both point sets. The Generalized Robust Point Matching (G-RPM) algorithm (Lin et al., 2003) derived from the TPS-RPM algorithm presented an alternative to solve the problem of outliers in both point sets by incorporating the curvature information. Although the G-RPM algorithm offered a double-sided-outlier handler, it did not provide a rigorous proof to show the equivalence between the original formulation of point matching problem and the energy function of the double-sided-outlier handler. In this paper we will review the TPS-RPM algorithm and demonstrate from a theoretical point of view its inherent limited performance when outliers are present in both point sets simultaneously. We will also present a double-sided outlier handling approach to overcome this limitation with a rigorous mathematical proof as the underlying theoretical support. This double-sided outlier handling approach is proved to be equivalent to the original formulation of the point matching problem. For a practical application, we also extend the TPS-RPM algorithm to non-rigid image registration by registering two sets of sparse

* Tel.: +1 713 792 2814; fax: +1 713 563 2479.
E-mail addresses: jyang4@mdanderson.org, jinzhong.yang@ieee.org

0167-8655/$ - see front matter
doi:10.1016/j.patrec.2011.01.015

features extracted from two images. The intensity information of the extracted features are incorporated to regularize the overall energy function of TPS-RPM algorithm in order to reduce the impact from outliers, which are resulted from automatic feature extraction, the common case in most real applications. Our experiments demonstrate the double-sided outlier handling approach and the efficiency of intensity information in assisting outlier detection.

The remainder of this paper is organized as follows. A review of the TPS-RPM algorithm and the analysis of its outlier handling function are given in Section 2. In Section 3 we provide a mathematical proof to demonstrate a double-sided outlier handling approach. Section 4 extends the TPS-RPM algorithm to image registration by incorporating intensity information. Experimental results for both point matching and image registration are provided in Section 5. Finally, we conclude our work in Section 6.

2. Review of the TPS-RPM algorithm

Given two point sets, the moving point set $X = \{\boldsymbol{x}_i : i = 1,2,\ldots,L\}$ and the fixed point set $Y = \{\boldsymbol{y}_j : j = 1,2,\ldots,N\}$ ($X, Y \in \Re^2$ for the sake of simplicity in presentation), the TPS-RPM algorithm detects the correspondence between X and Y and match these two point sets according to a smooth non-rigid transformation. To describe the algorithm, we first introduce the notation for the correspondence matrix and the spatial transformation. A binary *correspondence matrix* $\boldsymbol{P}$ with dimension $(L+1)\times(N+1)$ is defined to characterize the correspondence between X and Y,

$$\boldsymbol{P} = \left(\begin{array}{ccc|c} p_{11} & \cdots & p_{1N} & p_{1,N+1} \\ \vdots & \ddots & \vdots & \vdots \\ p_{L1} & \cdots & p_{LN} & p_{L,N+1} \\ \hline p_{L+1,1} & \cdots & p_{L+1,N} & 0 \end{array}\right). \tag{1}$$

The matrix $\boldsymbol{P}$ consists of two parts. The $L \times N$ inner submatrix defines the correspondence between X and Y. If $\boldsymbol{x}_i$ corresponds to $\boldsymbol{y}_j$, then $p_{ij} = 1$, otherwise $p_{ij} = 0$. The $(N+1)$th column and the $(L+1)$th row define the outliers in X and Y, respectively. If $\boldsymbol{x}_i$ (or $\boldsymbol{y}_j$) is an outlier, then $p_{i,N+1} = 1$ (or $p_{L+1,j} = 1$). $\boldsymbol{P}$ satisfies the row and column normalization conditions, $\sum_{j=1}^{N+1} p_{ij} = 1$, for $i = 1,2,\ldots,L$, and $\sum_{i=1}^{L+1} p_{ij} = 1$, for $j = 1,2,\ldots,N$. Let f denote a non-rigid *spatial transformation* that is used to characterize the non-rigid transform between X and Y. Under the mapping f, the point set X is transformed to $X' = \{\boldsymbol{x}'_i = f(\boldsymbol{x}_i) : i = 1,2,\ldots,L\}$.

2.1. Objective function

The TPS-RPM algorithm minimizes the following energy function for point matching,

$$[\hat{\boldsymbol{P}}, \hat{f}] = \arg\min_{\boldsymbol{P},f} E(\boldsymbol{P},f),$$
$$E(\boldsymbol{P},f) = \sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}\|\boldsymbol{y}_j - f(\boldsymbol{x}_i)\|^2 + \lambda\|\mathcal{L}f\|^2 - \zeta\sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}, \tag{2}$$

subject to the following constraints,

$$\begin{cases} p_{ij} \in \{0,1\}, & \text{for } i = 1,2,\ldots,L+1;\ j = 1,2,\ldots,N+1. \\ \sum_{j=1}^{N+1} p_{ij} = 1, & \text{for } i = 1,2,\ldots,L. \\ \sum_{i=1}^{L+1} p_{ij} = 1, & \text{for } j = 1,2,\ldots,N. \end{cases} \tag{3}$$

Here $\mathcal{L}$ in (2) is an operator denoting the smoothness regularization. Specifically, the thin-plate splines (TPS) are applied here,

$$\|\mathcal{L}f\|^2 = \int\int \left[\left(\frac{\partial^2 f}{\partial u^2}\right)^2 + 2\left(\frac{\partial^2 f}{\partial u \partial v}\right)^2 + \left(\frac{\partial^2 f}{\partial v^2}\right)^2\right] du\,dv, \tag{4}$$

where u and v represent the two dimensions of the points. The last term in (2) is designed to prevent excessive outlier rejection. λ and ζ are the weighting parameters to balance these terms.

Solving objective function (2) results into two coupled optimization problems: one is the correspondence problem, which is cast as a discrete linear assignment problem (Papadimitriou and Steiglitz, 1998), and the other is the transformation problem, which boils down to a least-square continuous optimization problem. The TPS-RPM algorithm adopts the softassign and deterministic annealing techniques to convert the discrete correspondence problem to a continuous one and thus achieves a robust point matching. The basic idea of softassign (Rangarajan et al., 1997) is to relax the binary correspondence matrix $\boldsymbol{P}$ to take values from interval $[0,1]$, while the row and column normalization constraints are enforced via the iterated row and column normalization method (Sinkhorn, 1964). The deterministic annealing technique (Yuille and Kosowsky, 1994) is introduced to control the behavior of the fuzzy correspondence. This technique is a heuristic continuation method that attempts to find the global optimum of the energy function at high temperature and tracks it as the temperature decreases. An entropy term, $T\sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}\log p_{ij}$, is added to the energy function (2) for this purpose. Here T is called the temperature parameter, which decreases gradually during optimization. This entropy term enables the fuzzy correspondence matrix to improve gradually and continuously during the optimization process instead of jumping around in the space of binary permutation matrices and outliers. When T reaches zero, $\boldsymbol{P}$ becomes binary and the outliers are identified naturally from the matrix (1). The objective function with a fuzzy correspondence matrix (to simplify notation, we use $\boldsymbol{P}$ as well) is formulated as

$$E(\boldsymbol{P},f) = \sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}\|\boldsymbol{y}_j - f(\boldsymbol{x}_i)\|^2 + \lambda\|\mathcal{L}f\|^2 + T\sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}\log p_{ij} - \zeta\sum_{j=1}^{N}\sum_{i=1}^{L} p_{ij}, \tag{5}$$

subject to

$$\begin{cases} 0 \leqslant p_{ij} \leqslant 1, & \text{for } i = 1,2,\ldots,L+1;\ j = 1,2,\ldots,N+1. \\ \sum_{j=1}^{N+1} p_{ij} = 1, & \text{for } i = 1,2,\ldots,L. \\ \sum_{i=1}^{L+1} p_{ij} = 1, & \text{for } j = 1,2,\ldots,N. \end{cases} \tag{6}$$

2.2. The TPS-RPM algorithm

The TPS-RPM algorithm solves the optimization problem in (5) and (6) as follow, which is similar to the EM algorithm involving a dual update process embedded within an annealing scheme. The two-step update is briefly summarized here.

- **Step 1:** Update the correspondence $\boldsymbol{P}$ by fixing the transformation f. For correspondence points $i = 1,2,\ldots,L$ and $j = 1,2,\ldots,N$,

$$p_{ij} = \frac{1}{T}\exp\left[\frac{\zeta}{T} - \frac{(\boldsymbol{y}_j - f(\boldsymbol{x}_i))^T(\boldsymbol{y}_j - f(\boldsymbol{x}_i))}{T}\right] \tag{7}$$

and for outliers possibilities in X, $i = 1,2,\ldots,L$ and $j = N+1$,

$$p_{i,N+1} = \frac{1}{T_0}\exp\left[-\frac{(\boldsymbol{y}_{N+1} - f(\boldsymbol{x}_i))^T(\boldsymbol{y}_{N+1} - f(\boldsymbol{x}_i))}{T_0}\right] \tag{8}$$