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Decomposition of mixed pixels based on bayesian self-organizing map and Gaussian mixture model

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ABSTRACT

How to decompose the mixed pixels precisely and effectively for multispectral/hyperspectral remote sensing images is a critical issue for the quantitative remote sensing research. This paper proposes a new method for decomposition of mixed pixels of multispectral/hyperspectral remote sensing images. The proposed method introduces the algorithm of Bayesian self-organizing map (BSOM) into the problem of the decomposition of mixed pixels. It estimates Gaussian parameters by minimizing the Kullback–Leibler information metric, and finishes the unmixing with Gaussian mixture model (GMM). In order to obtain a high unmixing precision, we need to extend the range of Gaussian distributions, and thus we propose a 3σ variance adjustment method to solve this problem. In addition, the proposed unmixing model automatically satisfies two constraints which are demanded for the problem of the decomposition of mixed pixels: abundances non-negative constraint (ANC) and abundances summed-to-one constraint (ASC). Experimental results on simulated and practical remote sensing images demonstrate that the proposed method can get good unmixing results for the decomposition of mixed pixels and is more robust to noise than other methods.

1. Introduction

For the limited space resolution of remote sensing images, every pixel of these images usually represents a large area on the ground. Thus, it is very possible that a pixel is a mixture of some typical ground objects (endmembers) in proportions (*abundance fractions*) (Plaza et al., 2004). The existence of mixed pixels reduces the accuracy of recognition and classification of ground objects based on pixel-level, and handicaps the development of the quantitative remote sensing technology. Moreover, a higher resolution will cause a higher cost for both initial acquisition and subsequent processing, thus an alternative way is to do the decomposition of mixed pixels. In practice, how to decompose these mixed pixels to obtain corresponding endmembers and abundance fractions precisely and effectively is an essential issue for high-accuracy classification and recognition of ground objects (Lu et al., 2003). Some different algorithms have been proposed to estimate the endmembers and abundance fractions including Linear Spectrum Mixture Model (LSMM) (Small, 2001; Haertel and Shimabukuro, 2005), neural network algorithms (Foody et al., 1997; Wang and Zhang, 1998; Zhang and Shao, 2002), and Fuzzy C-means clustering (Foody, 2000; Bastin, 1997; Friedman and Kandel, 1999), etc.

LSMM (Small, 2001; Haertel and Shimabukuro, 2005) was a widely used model to solve the unmixing problem. In this model, every mixed pixel is supposed to be a linear mixture of endmembers, and a linear equation group needs to be solved. Then a method named constrained least squares (CLS) (Shimabukuro and Smith, 1991) was proposed to solve the linear equation group with the abundances summed-to-one constraint (ASC). But this CLS method cannot satisfy both the ASC and the abundances non-negative constraint (ANC) automatically. It needs a forcible regulation (they call it quadratic programming) to make the ANC to be satisfied, and this forcible regulation may cause its unmixing result unacceptable. Except for the linear mixture model, there are some other models proposed for unmixing, such as the neural network model and the fuzzy model. Some researchers have applied the artificial neural network (ANN) (Foody et al., 1997; Wang and Zhang, 1998; Zhang and Shao, 2002) such as backward propagation (BP) neural network and radial-basis function (RBF) neural network to estimate the abundance fractions of mixed pixels. In these methods, the neural network adjusts its weight values by supervised training, and every training sample composes of a mixed pixel and its corresponding abundance fractions which have already been known. After supervised training, for every mixed pixel as the input, the outputs of the neural network are the unmixing abundance fractions. But this kind of methods cannot satisfy the ANC and the ASC, and that usually causes their unmixing results unacceptable. The fuzzy model based on Fuzzy C-means (FCM)

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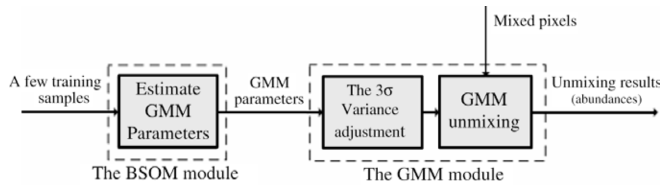

Fig. 1. Principle diagram.

clustering is also introduced (Foody, 2000; Bastin, 1997; Friedman and Kandel, 1999). In the fuzzy model, they generate the clustering centers and the membership degrees by unsupervised clustering, where the clustering centers correspond to endmembers, and the membership degrees correspond to abundance fractions. But this method is comparatively slow and not precise especially when the total number of data samples is large. Compared with the models mentioned above, here, we propose a probability model for unmixing the mixed pixels, which is similar to the fuzzy model to some extent, but is more elaborate than it. The comparison between the proposed probability model and the fuzzy model will be detailed in the following sections.

In this paper, we firstly introduce Bayesian self-organizing map (BSOM) into the problem of the decomposition of mixed pixels, and combine BSOM with Gaussian mixture model (GMM) to propose a new unmixing algorithm. This algorithm contains two modules: BSOM module and GMM module. The BSOM module is responsible for estimating GMM parameters by minimizing the Kullback–Leibler information metric. Because the chosen training samples for parameters estimation in the BSOM module are always comparatively pure, the estimated variances are so small to bring about large unmixing errors. To solve this problem, we propose a variance adjustment method based on the Gaussian probability distribution rule (we call it "3σ method"). In the GMM module, we first adjust the estimated variances by extending the range of Gaussian distributions with the "3σ method", and then do the unmixing with the estimated and adjusted GMM parameters. The principle diagram of our method is shown in Fig. 1.

The remainder of this paper is organized as follows: Section 2 introduces the BSOM algorithm which will be used in our method. Section 3 details the proposed unmixing method which is based on BSOM and GMM. In Section 4, some experimental results on both simulated data and real hyperspectral remote sensing data are shown. Conclusions are given in Section 5.

2. Bayesian self-organizing map

BSOM (Yin and Allinson, 2001; Yin and Allinson, 2001) proposed recently is an algorithm which can be used to estimate the parameters of GMM (Zhuang et al., 1996; Zhou and Wang, 2006).

GMM uses the mixture of some Gaussian probability density distributions to simulate the probability density curve with any shape. We call every Gaussian probability density distribution a Gaussian component, and suppose that there are K Gaussian components mixed in GMM, so the total probability of the data sample $\vec{x}$ is

$$p(\vec{x}) = \sum\nolimits_{i=1}^{K} p(\vec{x}|\vec{m}_{i,i})p(c_i), \tag{1}$$

where $\vec{m}_i$, $\sum_i$ and $p(c_i)$ are the mean vector, covariance matrix, and prior probability of the ith Gaussian component, respectively. And the conditional probability of $\vec{x}$ in the ith Gaussian component is

$$p(\vec{x}|\vec{m}_i, \sum_i) = \frac{1}{(2\pi)^{d/2}|\sum_i|^{1/2}} \exp\left\{-\frac{1}{2}(\vec{x}-\vec{m}_i)^{\mathrm{T}}\sum\nolimits^{-1}(\vec{x}-\vec{m}_i)\right\}, \tag{2}$$

where d is the dimension of $\vec{x}$. If we suppose $\sum_i = \mathrm{diag}(\sigma_i^2, \sigma_i^2, \ldots, \sigma_i^2) = \sigma_i^2 I$, then

$$p(\vec{x}|\vec{m}_i, \sigma_i) = \frac{1}{(2\pi)^{d/2}\sigma_i} \exp\left\{-\frac{(\vec{x}-\vec{m}_i)^{\mathrm{T}}(\vec{x}-\vec{m}_i)}{2\sigma_i^2}\right\}. \tag{3}$$

According to the Bayesian formula, the posterior probability can be described as

$$p(c_i|\vec{x}, \hat{\theta}_i) = \frac{p(\vec{x}|\hat{\theta}_i)\hat{p}(c_i)}{\sum_{i=1}^{K} p(\vec{x}|\hat{\theta}_i)\hat{p}(c_i)} = \frac{p(\vec{x}|\hat{\theta}_i)\hat{p}(c_i)}{\hat{p}(\vec{x})}, \tag{4}$$

where $\hat{\theta}_i = [\hat{\vec{m}}_i, \hat{\sigma}_i]$ and $\hat{p}(c_i)$ are estimated GMM parameters and

$$\sum\nolimits_{i=1}^{K} p(c_i|\vec{x}, \hat{\theta}_i) = 1, \tag{5}$$

$$p(c_i|\vec{x}, \hat{\theta}_i) \in [0, 1]. \tag{6}$$

To estimate GMM parameters, we introduce the BSOM algorithm which performs better than the traditional expectation maximize (EM) algorithm (Moon, 1996; Bilmes, 1998) at both convergence speed and escape of local minimum. The BSOM algorithm estimates GMM parameters by minimizing the Kullback–Leibler information metric defined as

$$I = -\int \ln\left[\frac{\hat{p}(\vec{x})}{p(\vec{x})}\right] p(\vec{x})\,\mathrm{d}\vec{x}. \tag{7}$$

For the parameter estimation of probability model, the Kullback–Leibler information metric represents the average information which resides in every training data sample after estimation. So it is always non-negative, and equates to 0 only if the estimated model accords with the real model perfectly, which is only an ideal circumstance.

By minimizing I, we get updating formulas of the BSOM algorithm as follows:

$$\hat{\vec{m}}_i(n+1) = \hat{\vec{m}}_i(n) + \alpha(n)p(c_i|\vec{x}(n), \hat{\theta}_i)(\vec{x}(n) - \hat{\vec{m}}_i(n)), \tag{8}$$

$$\hat{\sigma}_i^2(n+1) = \hat{\sigma}_i^2(n) + \alpha(n)p(c_i|\vec{x}(n), \hat{\theta}_i) \times \{[\vec{x}(n) - \hat{\vec{m}}_i(n)]^{\mathrm{T}}[\vec{x}(n) - \hat{\vec{m}}_i(n)] - \hat{\sigma}_i^2(n)\}, \tag{9}$$

$$\hat{p}_i(n+1) = \hat{p}_i(n) + \alpha(n)\{p(c_i|\vec{x}(n), \hat{\theta}_i) - \hat{p}_i(n)\}, \tag{10}$$

(8) and (9) for $i \in \eta_r$, and (10) for all i, where $\alpha(n)$ is the instantaneous learning rate and η_r is the updating area similar to the neighborhood area of the self-organizing map (SOM) neural network (Guo and Forster, 1994; Lee and Lathrop, 1992). Here, the neighborhood function in the SOM is replaced by the posterior probability $p(c_i|\vec{x}, \hat{\theta}_i)$. So, for the BSOM network, every node composes of a mean vector, a variance and a prior probability, while the SOM network represents its node only by a vector. The details of the mathematical derivation about the formulas (8)–(10) can be found in the Appendix.

For every input training sample $\vec{x}$, we judge the node with highest posterior probability to be the winner node. Then we update the mean vectors and the variances of the nodes in updating area η_r according to (8) and (9), and update the prior probabilities of all nodes according to (10) to satisfy the constraint described as follow:

$$\sum_{i=1}^{K} \hat{p}(c_i) = 1. \tag{11}$$

All nodes updating according to (8)–(10) is feasible, and moreover, the introduction of η_r can reduce computation time very much.

3. The proposed unmixing method

3.1. The unmixing scheme based on BSOM and GMM

Our unmixing algorithm is based on the assumption that the data samples have a probability distribution described by GMM. Suppose