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Evaluating Harker and O'Leary's distance approximation for ellipse fitting

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Abstract

Harker and O'Leary's [Harker, M., O'Leary, P., 2006. First order geometric distance (the myth of Sampsonus). British Machine Vision Conf., 87–96] recently proposed a new distance measure for conics. This paper compares its accuracy and effectiveness against several other error of fits (EOFs) for ellipses using: (1) visualisations of the distortions with respect to the Euclidean distance; (2) a set of evaluation measures specifically designed for assessing ellipse EOFs [Rosin, P.L., 1996a. Analysing error of fit functions for ellipses. Pattern Recognition Lett. 17, 1461–1470; Rosin, P.L., 1996b. Assessing error of fit functions for ellipses. Graphical Models Image Process. 58, 494–502]; (3) the accuracy of LMedS ellipse fitting using the various EOFs.

Keywords: Distance approximation; Ellipse fitting; Evaluation

1. Introduction

Conic fitting, and in particular ellipse fitting, has been found to be an extremely useful tool in computer vision, with many applications such as: face detection (Sun et al., 1998), gaze determination (Wang et al., 2005), camera calibration (Heikkilä, 2000), shape measurement (Rosin, 2003), and the analysis of grain (Shashidhar et al., 1997), potatoes (Zhou et al., 1998), sperm heads (Park et al., 1997), etc.

There are two strands of research in ellipse fitting. The first considers alternative frameworks for performing the fitting, e.g. applying robust statistics to reduce the effects of outliers (Rosin, 1999; Roth and Levine, 1993), the use of various optimisation tools (e.g. genetic algorithms, Roth and Levine, 1994), constraining the fitted conic to be an ellipse (Fitzgibbon et al., 1999), constrained multiple fits (e.g. concentric ellipses) (O'Leary et al., 2005), and so on. The second strand focuses on the objective function, i.e. the error of fit (EOF); since the true Euclidean shortest distance between a point and an ellipse requires solving a quartic equation this is usually replaced by simpler and more efficient approximations. The most common is the so-called *algebraic distance*. Given the implicit equation of a conic

$$Q(x,y) = ax^2 + bxy + cy^2 + dx + ey + f,$$

then the algebraic distance from the point $\boldsymbol{P}_i = (x_i, y_i)$ to the conic is defined directly from the above as

$$\mathrm{EOF}_1 = Q(x_i, y_i) = ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f.$$

An often used refinement of the algebraic distance suggested by Sampson (1982) is to inversely weight it by its gradient

$$\mathrm{EOF}_2 = \frac{Q(x_i, y_i)}{|\nabla Q(x_i, y_i)|}.$$

In fact, many distance approximations have been developed in recent years; 13 are described and compared in

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(Rosin, 1996a,b). See also Fitzgibbon and Fisher (1995) for experimental comparison between various distance measures and fitting constraints.

Recently, Sampson's approach was revisited by Harker and O'Leary (2006) who noted that Sampson provided the *distance to the first order approximation of the Euclidean distance*, while they developed the *first order approximation to the distance function*. This results in a more complicated expression which, when simplified (without loss of generality) by translating the conic such that $\mathbf{P}_i$ lies at the origin, yields

$$\mathrm{EOF_{HO}} = \sqrt{\left|-\frac{w_n}{w_d} f^2\right|},$$

where

$$\begin{aligned} w_n &= d^4 + e^4 - 32acf^2 + 16a^2f^2 + 16b^2f^2 + 16c^2f^2 \\ &\quad - 16bdef + 2d^2e^2 + 8cd^2f + 8ae^2f - 8ad^2f - 8ce^2f, \\ w_d &= d^6 - e^6 - 3d^4e^2 - 3d^2e^4 + 10ad^4f + 10ce^4f - 8cd^4f \\ &\quad - 8ae^4f + 18bd^3ef + 18bde^3f + 32a^3f^3 + 32c^3f^3 \\ &\quad + 32ab^2f^3 - 32ac^2f^3 - 32a^2cf^3 + 32b^2cf^3 \\ &\quad - 32a^2d^2f^2 - 32c^2e^2f^2 - 20b^2e^2f^2 - 20b^2d^2f^2 \\ &\quad + 40acd^2f^2 + 40ace^2f^2 - 8c^2d^2f^2 - 8a^2e^2f^2 \\ &\quad + 2ad^2e^2f + 2cd^2e^2f - 24abdef^2 - 24bcdef^2. \end{aligned}$$

Although not explicitly stated in (Harker and O'Leary, 2006), the $-\frac{w_n}{w_d}f^2$ term is not always non-negative.

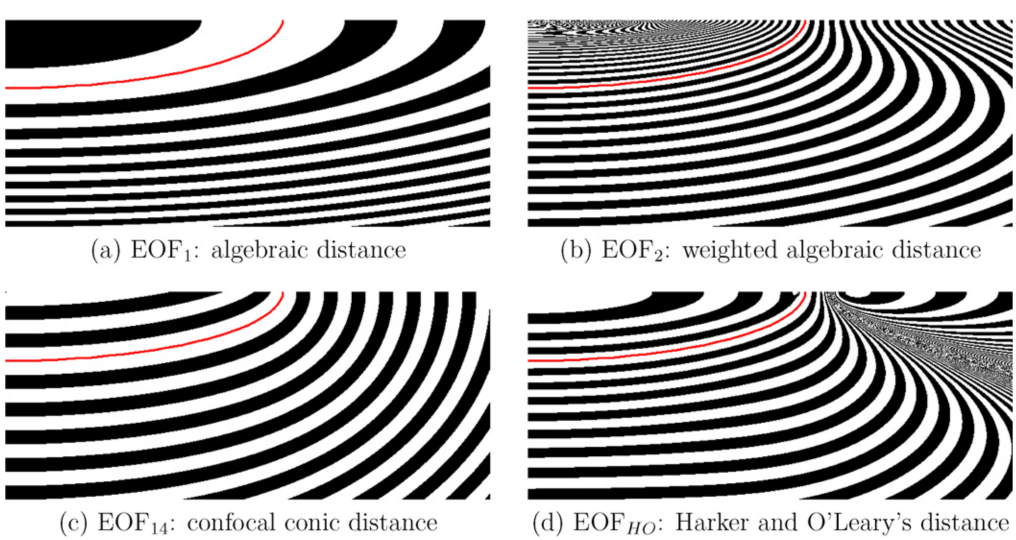
In their paper Harker and O'Leary (2006) compared their distance approximation against Sampson's gradient weighted algebraic distance. However, since there are many other distance approximations available, some of which are both relatively simple and accurate, it is of interest to compare their measure against some of these.

In particular, we compare their method to one which was based on the ellipse and its confocal hyperbola that passes through $\mathbf{P}_i$ (Rosin, 1998). Such confocal conics are mutually orthogonal, and since much of the hyperbola is relatively straight it is a good approximation to the normal from the point to the ellipse. Computing the distance is then straightforward, and details are provided in (Rosin, 1998).

Previously, comparisons were carried out on the above confocal conic error of fit (denoted by EOF_{14}) and the 13 described in (Rosin, 1996a,b), and the former (EOF_{14}) was shown to perform the best. In the same manner, the analysis is shown here for the new distance $\mathrm{EOF_{HO}}$ and is duplicated for EOF_1, EOF_2, and EOF_{14} for easy reference.

First, to visualise distortions in the approximate distance function, blocks of values over a fixed range are alternately coloured black and white; see Fig. 1. A more elongated ellipse than in (Harker and O'Leary, 2006) is used, as this makes the distortions more apparent. Although $\mathrm{EOF_{HO}}$ is well behaved near the ellipse it can be seen to display substantial distortions at greater distances, particularly outside the ellipse (as compared to Sampson's EOF_2 which has severe distortions inside the ellipse). In comparison, the confocal conic distance has no singularities, and is overall better behaved.

Next, a more quantitative analysis was carried out, using the methods specifically designed in (Rosin, 1996a,b) for assessing error of fit functions for ellipses. Four basic terms were defined, quantifying the linearity of a distance approximation w.r.t. the Euclidean distance (computed by the Pearson correlation coefficient), curvature bias (how the distances vary as a function of the ellipse's curvature), asymmetry (between distances inside and outside the ellipse), and the overall goodness (denoted by G) of a distance approximation w.r.t. the Euclidean



(a) EOF_1: algebraic distance
(b) EOF_2: weighted algebraic distance
(c) EOF_{14}: confocal conic distance
(d) EOF_{HO}: Harker and O'Leary's distance

Fig. 1. Iso-distance bars of distance approximations to the ellipse (red in the web version) which has aspect ratio = 4.