Signal Processing: *Image Communication* 41 (2016) 115–127

Contents lists available at ScienceDirect

Signal Processing: *Image Communication*

journal homepage: www.elsevier.com/locate/image

A relaxation approach to computation of second-order wedgelet transform with application to image compression

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ARTICLE INFO

Article history:
Received 29 May 2015
Received in revised form
22 November 2015
Accepted 22 December 2015
Available online 31 December 2015

Keywords:
Fast second-order wedgelet transform
Non-linear least squares
Quad-tree pruning
Rate-distortion analysis
Multiresolution image compression
Disparity images

ABSTRACT

Wedgelet approximation of an image block is classically obtained by an exhaustive search in a predefined dictionary where representation error of all basis elements is examined. Although this strategy leads to the selection of the best atom, due to intolerable computational complexity, the practical applications of classical wedgelet transform is severely limited. In this paper, by employing non-linear least squares and discontinuity relaxation, we suggest an iterative estimation procedure to speed up computation of the second-order wedgelet transform. Accuracy and speed of this wedgelet computation approach is studied by an image compression framework which utilizes both wedgelets and platelets to approximate image blocks. The **M**-term approximation of second-order wedgelets and second-order platelets is studied in this framework and is concluded to be $O(M^{-3})$. Our proposed wedgelet computation surpasses the state of the art *Moments-based Second-order Wedgelet Transform* in accuracy while achieving roughly the same speed. It also achieved noticeable quality improvement over JPEG2000 in compression of disparity images (5 dB at 0.15 bpp).

1. Introduction

Multiresolution transforms have been intensively exploited to represent, approximate or compress images. In this area of research, classical separable wavelets which sparsely represent point singularities are gradually accompanied by more recent geometrical wavelets which sparsely represent discontinuities along curves. Modern geometrical wavelets not only determine location and scale of the image edges, but also consider their orientation, thickness, or curvature. These representations fall into two broad categories: *adaptive* and *non-adaptive* geometrical wavelets. Non-adaptive geometrical wavelets approximate an image as a whole using a linear combination of all basis functions in a set which usually form a *tight frame*. In contrast, adaptive geometrical wavelets create an adaptive partitioning of the image and represent image blocks by picking up a subset of atoms from a predefined dictionary, and hence sometimes they are also called *geometric tilings*. While this fact gives them the opportunity to better capture geometry in the image, they usually rely on time consuming search for selecting proper atoms.

Adaptive geometrical wavelets in two dimensional domain construct an adaptive quadtree partitioning to produce multiresolution representation of the image structure, and are often defined based on the dictionary of beamlets. A discrete *first-order beamlet* represents a straight line segment in the image block with its endpoints lying on the boundary pixels of the block [3]. A discrete *second-order beamlet* represents a bent segment and

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http://dx.doi.org/10.1016/j.image.2015.12.005
0923-5965/

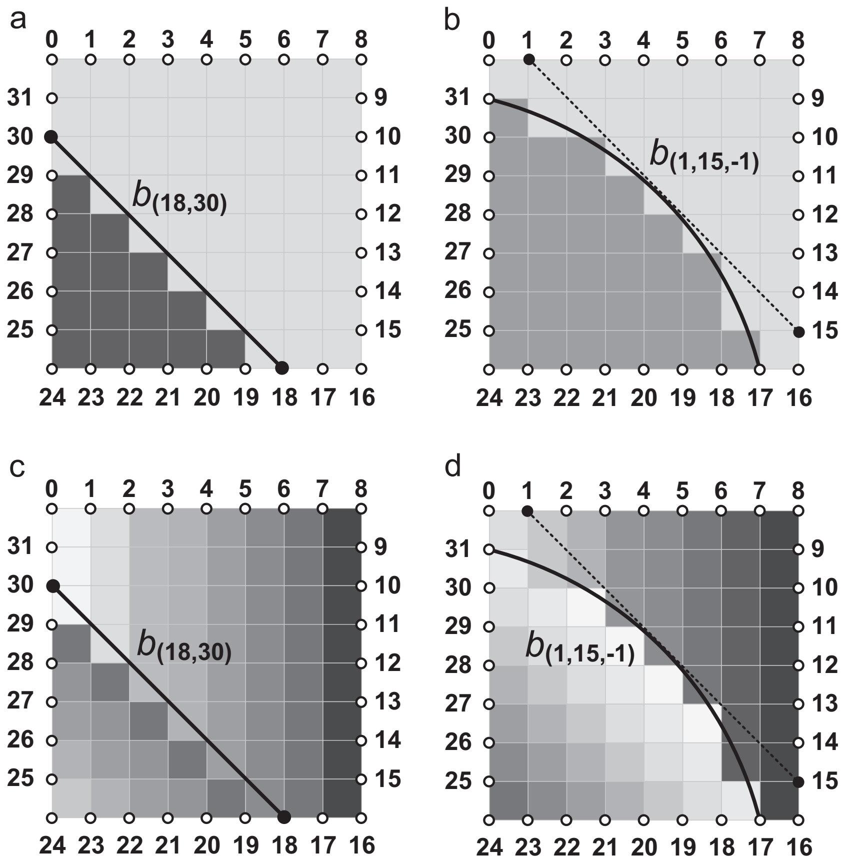

Fig. 1. Sample approximations of an image block superimposed by their corresponding beamlets and obtained by (a) a first-order wedgelet, (b) a second-order wedgelet, (c) a first-order platelet, (d) a second-order platelet.

generalizes the first-order beamlet by considering an integer curvature which measures its deviation from the straight line segment [6,7]. Beamlets regardless of their order, decompose the block into two regions.

The most widely used geometrical wavelet is a *first-order wedgelet* [4] which represents a nearly straight sharp discontinuity in the image block by decorating a first-order beamlet with two intensity values which approximate the block as two constant regions [8], as depicted in Fig. 1(a). Later, first-order wedgelets are generalized by more complex discontinuity and surface models. As shown in Fig. 1(b), *second-order wedgelets* represent curved edges in the image by second-order beamlets defined usually as quadratic polynomials [6,7]. *Platelets* describe each region formed by the beamlet in the image block using a linear function [15]. Examples of first-order platelets and second-order platelets are shown in Fig. 1(c) and (d), respectively. A generalization of wedgelets to dimensions higher than two has been also studied and is called *surflets* [1]. Another generalization of wedgelet is *smoothlet* which considers edge blur [9].

Wedgelets are shown to be asymptotically optimal representation for approximating piecewise constant images which contain sharp edges [4]. It is well-known that in representation and compression of natural images, such geometric tilings have to be combined with classical wavelet-based coding techniques to compete with state of the art textured image compression algorithms such as JPEG2000 [14,1]. Nevertheless, the practical applications of wedgelets have usually been limited because of time-consuming computations.

In *classical Wedgelet Transform (WT)*, beamlet parameters are determined by an exhaustive search procedure which assesses every possible atom and is therefore a prohibitively slow process. In an $N \times N$ image, computational complexity of this exhaustive search is $O(N^4 \log_2 N)$ for first-order wedgelets [4,8]; and it remains the same for the case of *classical Second-order Wedgelet Transform (SWT)* provided that the maximum quantized curvature value is considered constant and therefore is not growing with respect to N [9]. Therefore, the exhaustive search restricts the application of adaptive geometrical wavelets in the real world scenarios. There exist several fast replacements in the literature which usually cause a substantial degradation in approximation quality. As a result, a better balance between computation time and approximation quality is still desirable.

In order to avoid searching for beamlet parameters in first-order wedgelet transform and yet attain a reasonable quality, a moments-based estimation approach is advertised in Lisowska [8] based on the prior work of Popovici in Popovici and Withers [12] and further extended to second-