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Applying the Hough transform pseudo-linearity property to improve computing speed

E. Duquenoy [a], A. Taleb-Ahmed [b,*]

[a] LEMCEL, Université du Littoral - Côte d'Opale, 50, Rue Ferdinand Buisson, BP 717, 62228 Calais Cedex, France

[b] LAMIH UMR CNRS 8530, Université de Valenciennes et du Hainaut Cambrésis, Le mont Houy, 59313 Valenciennes Cedex 9, France

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Abstract

This work describes a general method of acceleration of the convergence of the Hough transform based, on the one hand, on an improvement of the *image analysis* speed, and, on the other hand, on the space undersampling of the image. This method is used in image processing to extract lines, circles, ellipses or arbitrary shapes. The results presented are applied to the detection of straight-line segments and ellipses, but can be extended to any type of transform.

Keywords: Hough transform; Space undersampling; Speed optimisation; Pseudo-linearity property; Peak detection; Straight-line segment detection; Ellipse center detection

1. Introduction

The Hough transform (HT) is a method for detecting analytically-described shapes, including straight lines, circles and ellipses (please refer to Leavers (1993) for a synthetic presentation of the Hough transform). Under certain conditions, the Hough transform also permits the recognition of any shape, whether it has been described analytically or not (Sakai et al., 1996; Kim et al., 2001; Achalakul and Madarasmi, 2002). In fact, the method is not limited to detecting the objects cited above, but rather can be applied to a wide number of activities, such as motion detection (Kalviainen, 1993), temporal signal monitoring (Imiya, 1996), chirp detection (Sun and Willett, 2001) and character recognition (Shiku et al., 1996).

The Hough transform works to determine the geometric parameters of a shape via a voting procedure. Every point in the image containing the shape votes for one or several points of the parameter space. The dimensions of this space depend on the type of shape desired: thus, a two-dimensional space will be needed to search for a straight line, whereas a three-dimensional space will be needed to search for a circle. Thus, every point in the initial image votes for the parameters of a shape that is likely to cross through that point, which implies that an infinite number of shapes can pass through a single point. However, because the parameter space is discrete, the number of shapes is finite.

Duda and Hart (1972) proposed the following *one-to-many* transform method for detecting straight lines: for every point $M(x,y)$ of the initial image, and for θ variants from $-\pi$ to $+\pi$, the transform creates a corresponding curve equation, $\rho(\theta) = x\cdot\cos\theta + y\cdot\sin\theta$, via *normal parametrization*, which increments the content of the counter, or accumulators, that cross the coordinate parameter space (ρ,θ). Another approach to the voting process is the *many-to-one* transform. In this type of transform, an α-uplet of points votes for one parameter point, given a shape with α parameters.

* Corresponding author. Tel.: +33 327511334; fax: +33 327511316.
E-mail addresses: eric.duquenoy@univ-littoral.fr (E. Duquenoy), taleb@univ-valenciennes.fr (A. Taleb-Ahmed).

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Regardless of the approach used, the Hough transform is a very costly application in terms of computing time, and numerous proposals have been made to improve its speed.

To reduce the execution time of the Hough transform, the number of the points to be processed must also be reduced (Kiryati and Bruckstein, 1991). To accomplish this, Tsuji and Matsumoto (1978) and Yoo and Sethi (1993) use a priori information, such as the direction and amplitude of the vector gradient. On the other hand, Xu and Oja (1993), Bergen and Shvaytser (1991) and Kato et al. (2000) recommend combining a random choice of binary image points with the simultaneous detection of the parameter space maxima. However, such methods are quite sensitive to the noise in the image. This noise increases the number of points to be processed, which in turn increases the computing time and the number of erroneous detections.

In this paper, we present a general method for accelerating the convergence of the Hough transform by increasing the image analysis speed. Following this introduction, Section 2 shows how the Hough transform can be controlled using an algebra that, though basic, is sufficient to justify the structural choices for the software, or even the hardware. Section 3 details our procedure for undersampling the binary image. In Section 4, this undersampling is associated with an adaptive search for the parameter space maxima that allows the detection of this maxima to be anticipated. In Section 5, we present the performance results followed by our concluding remarks.

2. The pseudo-linearity of the Hough transform

2.1. The "one-to-many" or "one to m" transform

We consider that the Hough transform is a linear relation $\mathscr{HT}$ of the set I of active points in binary image to be processed (data point space) in a set P representing the parameter space. An element M of the set I is characterized by the vector of its $\underline{\boldsymbol{x}}$ co-ordinates of dimension n for the values in $\mathbb{R}^n$; whereas an element from the set P is characterized by parameters vector of $\underline{\boldsymbol{p}}$ of dimension m for the values in $\mathbb{R}^m$ and is associated with an accumulator in an accumulation space A.

The $\mathscr{HT}$ relation matches each element M_i of the set I, where $i \in \{1 \ldots \text{card} I\}$, with a curve $\mathscr{C}_i$ (i.e. a set of elements in the arrival set of P, which increments the crossed accumulators). The transform of two elements M_i and M_j in I, where $i \in \{1 \ldots \text{card} I\}$ and $M_i \neq M_j$ is defined as the sum of the two curves $\mathscr{C}_i$ and $\mathscr{C}_j$ in arrival set P. Given M_i and $M_j \in I$, if $I_i = \{M_i\}$ and $I_j = \{M_j\}$, it follows that:

$$\begin{cases} \mathscr{HT}(M_i) \equiv \mathscr{HT}(I_i) = \mathscr{C}_i \\ \mathscr{HT}(M_j) \equiv \mathscr{HT}(I_j) = \mathscr{C}_j \\ \mathscr{HT}(\{M_i, M_j\}) \equiv \mathscr{HT}(I_i \cup I_j) = \mathscr{C}_i + \mathscr{C}_j \end{cases} \\ \Rightarrow \mathscr{HT}(I_i \cup I_j) = \mathscr{HT}(I_i) + \mathscr{HT}(I_j) \tag{1}$$

Given the result (1), it is possible to decompose the transform of a subset of I (i.e. $I_i \cup I_j$) into the sum of the transforms of singletons composing this subset. By extension of this property and considering a partition $kP(I) = \{I_1 \cdots I_q\}$ such as $I = I_1 \cup I_2 \cup \cdots \cup I_q$, $\forall i$ and $\forall j \in \{1 \ldots q\}$ where $q \leqslant \text{card} I$, $i \neq j$ and $I_i \cap I_j = \emptyset$, it can be deduced that:

$$\mathscr{HT}(I) = \mathscr{HT}(I_1 \cup I_2 \ldots \cup I_q) \\ \Rightarrow \mathscr{HT}(I_1 \cup I_2 \cdots \cup I_q) = \sum_{k=1}^{q} \mathscr{HT}(I_k) \tag{2}$$

The result of Eq. (2) is stated as follows: *the Hough transform of a set is equal to the sum of the transforms of each of its disjoint subsets, both paired and complementary* (Duquenoy, 1998).

2.2. The "many-to-one" ("m-to-1") transform

When using a "many-to-one" approach to the Hough transformation process, the value of the parameters vector $\boldsymbol{p}$ in space P is calculated starting from an element M of the set $I \times I \cdots \times I$, making M a subset of elements that are distinct from I. Each one is characterized by the vector of its $\underline{x}$ coordinates. The initial space thus consists of elements taken from $I \times I \cdots \times I$, and the arrival space P remains unchanged. It is no longer necessary to calculate a curve $\mathscr{C}_i$ (the set of elements in arrival space P) but rather only a single element of this space.

The $\mathscr{HT}$ relation associates a point that has a parameters vector $\underline{\boldsymbol{p}}$ in arrival space P to each element $M = (M_1 \ldots M\gamma)$, in the set $I \times I \cdots \times I = I^\gamma$, where γ represents the dimension of the parameter space (i.e. the number of required parameters). Thus, the relation expressed in Eq. (2) always remains valid, provided that I^γ is considered as an initial set.

Given the "pseudo-linearity" property shown in Eq. (2), two steps are needed to calculate the Hough transform optimally:

1. The transform calculation of the initial binary image must be decomposed, so that the overall calculation is equal to the sum of the transforms of the different subsets of points.
2. Each transform of the decomposed subsets must be calculated independently, and then used in the method suggested in the following section.

3. Acceleration by undersampling

Formalism

The undersampling used in our method consists of decomposing the initial binary image I of dimensions $N * N$ into a set of under-images $I_{a,b}^{\mathscr{O}_e}$, where the parameter $\mathscr{O}_e$ is the undersampling order and the couple (a, b) the reference of the under-image. Respecting the conditions of the