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Raman scattering of a donor impurity in a quantum ring

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ABSTRACT

By using the matrix diagonalization method within the effective-mass approximation, we have investigated the electron Raman scattering process associated with a donor impurity confined by a quantum ring in the presence of a magnetic field perpendicular to the plane of the ring. With typical semiconducting *GaAs*-based materials, the differential cross section has been examined based on the computed energies and wave functions. The results show that the differential cross section of the electron Raman scattering of a donor impurity in a quantum ring is affected by the geometrical size of the ring, the confinement strength and the external magnetic field.

1. Introduction

Semiconductor quantum dots are one kind of useful quantum structures which can be fabricated by directly self-organized growth [1]. The three-dimensional quantum confinement of electrons drastically changes the electronic structure in quantum dots from that of bulk semiconductors with a continuous energy spectrum to that with essentially discrete energy levels. A quantum ring (QR) is a quantum dot structure with a 'hole' in its middle. Compared with quantum dots, QRs belong to another kind of topological structures in which more rich phenomena can be clearly shown [2,3]. In 2000, Lorke and co-worker applied self-assembly techniques to create *InGaAs* rings containing only a few electrons [3]. They first observed far-infrared optical response in QRs, revealing a magneto-induced change in the ground state from angular momentum $\ell = 0$ to $\ell = 1$, with a flux quantum piercing the interior. Using a droplet-epitaxial technique, Kuroda and co-worker investigated the optical transitions in QRs [4]. Quite different from the conventional submicron mesoscopic structures, the nanoscopic rings are in the true quantum limit. The weak electron–electron interaction in these rings makes them most suitable for the observed state transitions can be well explained with the single-electron spectrum of a parabolic ring [5]. In addition, the ringlike confinement breaks down the generalized Kohn theorem so that, unlike in quantum dots, the excitation spectrum of QRs may reveal Coulomb interaction effects [6].

On the other hand, the understanding of the donor impurity states in the confined systems is an important problem in semiconductor physics. In the optical transition of quantum confined few-electron systems, the analysis of the donor impurity states is also inevitable because the confinement of quasiparticles in such structure leads to the enhancement of the oscillator strength of electron-impurity excitations. Meanwhile, the dependence of the optical transition energy on the confinement strength allows the tunability of the resonance frequency. Hence, a number of theoretical investigations of a donor impurity in low-dimensional semiconductors have been published [7–17]. It is believed that a fundamental study on the properties of a donor impurity in semiconductor QRs is also important, because the dimensionality and the ring geometry often introduce unexpected physical phenomena.

Raman scattering in semiconductor structures was devised more than 30 years ago as a powerful tool for the identification of electronic excitations [18]. Raman scattering can provide the direct information on the electronic structure and optical properties of semiconductor nanostructures [19]. Hence, electron Raman scattering experiments are a powerful tool for the investigation of semiconductor nanostructures. On the other hand, the calculation of the differential cross section (DCS) of electron Raman scattering remains a rather interesting and fundamental issue to achieve a better understanding of semiconductor nanostructures. Experimental research on the electron Raman scattering in quantum dots has been reported [20–22]. In order to interpret experimental results, some authors have theoretically investigated the electron Raman scattering of quantum dots [23–26]. They found that the DCS of quantum dots strongly depends on the quantum dot size and the strength of the external field. To our knowledge, the electron Raman scattering of a donor impurity in QRs has not been investigated so far. In this work, we will study the electron Raman scattering of a donor impurity in a QR with a parabolic potential, and investigate the influences of the geometrical size of the ring, the confinement strength and the external magnetic field on the DCS.

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2. Theory

We model QRs as they are realized in the laboratory [2,3] by a potential of the form $V(r) = \frac{1}{2} m_e^* \omega_0^2 (r - r_0)^2$. Within the framework of effective-mass approximation, the Hamiltonian of a donor impurity confined by a QR when the applied magnetic field is perpendicular to the $x - y$ plane may be written as

$$H = \frac{1}{2m_e^*}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 + \frac{1}{2} m_e^* \omega_0^2 (r - r_0)^2 - \frac{e^2}{\epsilon r}, \tag{1}$$

where m_e^* is the effective mass of an electron, e is the electric charge of an electron, c is the speed of light, and ϵ is the dielectric constant. $\vec{r}(\vec{p})$ is the position vector (the momentum vector) of the electron originating from the center of the ring. The parameter r_0 is the radius of the QR and ω_0 defines the strength of the two-dimensional potential [3,5]. With the symmetric gauge for magnetic field $\vec{A} = (B/2)(-y, x, 0)$, the Hamiltonian then reads

$$H = \frac{p^2}{2m_e^*} + \frac{1}{2} m_e^* \omega^2 r^2 - m_e^* \omega_0^2 r r_0 + \frac{1}{2} m_e^* \omega_0^2 r_0^2 - \frac{1}{2} L_z \omega_c - \frac{e^2}{\epsilon r}, \tag{2}$$

where $\omega^2 = \omega_0^2 + \omega_c^2/4$, $\omega_c = eB/cm_e^*$ is the cyclotron frequency, and L_z is the z-component of the angular momentum. The Hamiltonian has cylindrical symmetry which implies the orbital momentum L is a conserved quantity, i.e., a good quantum number. Hence, the eigenstates of the donor impurity in a QR can be classified according to the orbital angular momentum L. To obtain the eigenfunction and eigenenergy associated with the donor impurity in a QR, the Hamiltonian is diagonalized in the model space spanned by two-dimensional harmonic states

$$\Psi_L = \sum_i c_i \phi_i^{\omega'}(\vec{r}), \tag{3}$$

where $\phi_i^{\omega'}(\vec{r}) = R_{n_i \ell_i}(r) \exp(-i\ell_i \theta)$ is ith eigen-state of the two-dimensional harmonic oscillator with a frequency ω' and an energy $(2n_i + |\ell_i| + 1)\hbar\omega' \cdot R_{n\ell}(r)$ is the radial wave function, given by

$$R_{n\ell}(r) = N \exp(-r^2/(2a^2)) r^{|\ell|} L_n^{|\ell|}(r^2/a^2), \tag{4}$$

in which N is the normalization constant, $L_n^{\ell}(x)$ is the associated Laguerre polynomial, $a = \sqrt{\hbar/(m_e \omega')}$ and $\hbar$ is the reduced Plank constant. The radial and orbital angular momentum quantum numbers can have the following values

$$n = 1, 2, \ldots, \quad \ell = 0, \pm 1, \pm 2, \ldots \tag{5}$$

Here ω' is an adjustable parameter, and is, in general, not equal to ω.

Let $N = 2n + \ell$. Let $\{\Psi_K\}$ denote the set of basis functions including all the Ψ_K having their N smaller or equal to an upper limit $N_{\max}$. It is obvious that the total number of basis functions of the set is determined by $N_{\max}$. After the diagonalization we obtain the eigenvalues and eigenvectors. Evidently, the eigenvalues depend on the adjustable parameter ω'. In our calculation, ω' serves as a variational parameter to minimize the low-lying state energy. The matrix diagonalization method consists in calculating the matrix elements with the given basis and extracting the lowest eigenvalues of the matrix generated. The better the basis describes the Hamiltonian, the faster the convergence will be.

The DCS of electron Raman scattering in a volume V, per unit solid angle, considering incoming light of frequency ω_l, and the scattering light of frequency ω_s is given by Ref. [27]

$$\frac{d^2\sigma}{d\Omega d\omega_s} = \frac{V^2 \omega_s^2 \eta(\omega_s)}{4\pi^2 c^4 \hbar \eta(\omega_l)} \sum_f |M|^2 \delta(E_f - E_i), \tag{6}$$

where $\eta(\omega)$ is the refraction index as a function of the radiation frequency. In Eq. (6), M and $\delta(E_f - E_i)$ are defined by

$$M = \sum_a \frac{\langle f|H_s|a\rangle\langle a|H_l|i\rangle}{E_i - E_a + i\Gamma_a} \tag{7}$$

and

$$\delta(E_f - E_i) = \frac{\Gamma_f}{\pi[(E_f - E_i)^2 + \Gamma_f^2]}. \tag{8}$$

Here, $|i\rangle, |a\rangle$ and $|f\rangle$ denote initial, intermediate and final states of the system with their corresponding energies E_i, E_a and E_f, respectively. These energies are determinable by using the matrix diagonalization method. And Γ_f is the life-time width. In the dipole approximation, the interaction with the incident radiation field is described by the Hamiltonians

$$H_l = \frac{|e|}{m_e}\sqrt{\frac{2\pi\hbar}{V\omega_l}}\vec{e}_l \cdot \vec{p}, \quad \vec{p} = -i\hbar\nabla, \tag{9}$$

where m_e is the free electron mass. The Hamiltonian of the interaction with the secondary radiation field is given by

$$H_s = \frac{|e|}{m_e^*}\sqrt{\frac{2\pi\hbar}{V\omega_s}}\vec{e}_s \cdot \vec{p}, \quad \vec{p} = -i\hbar\nabla. \tag{10}$$

Here $\vec{e}_l$ ($\vec{e}_s$) is the unit polarization vector for the incident (secondary) radiation.

3. Results and discussion

Our numerical computation is carried out for one of the typical semiconducting materials, *GaAs*, as an example with the material parameters shown in the following: $m_e^* = 0.067 m_e$, and $\epsilon = 12.4$ [28]. The lifetimes of the final and intermediate states are $\Gamma_f = \Gamma_a = 1$ meV. In this work, the DCS of electron Raman scattering for a three-level system is calculated for a QR using Eq. (6). We take the initial state as Ψ_0, and consider Raman excitations from this state. In the initial state we have an electron in the ground state of the conduction band and a incident photon of energy $\hbar\omega_l$. From the ground state the electron carries out a transition to the second intermediate state with energy E_a. From the second intermediate state the electron undergoes a transition toward the first excited state Ψ_1, emitting the laser radiation as secondary radiation of energy $\hbar\omega_s$. The intermediate states can, in principle, be taken to span the entire Hilbert space of eigenfunctions of the Hamiltonian given by Eq. (2). For simplicity, we restrict the intermediate state to the second excited state Ψ_2.

In Fig. 1, we set $\hbar\omega_0 = 10.0$ meV and plot the DCS of electron Raman scattering of the donor impurity in a QR as a function of the diffusion photon energy for five different ring radius values, i.e., r_0=0.0, 5.0, 10.0, 12.0 and 15.0 nm, respectively. In Fig. 1, the applied magnetic field is to be 0T, i.e., in the absence of external magnetic field. From this figure we can find that the geometrical size effect of the QR on the DCS is significant. We find that the all peak positions of DCS shift to lower energies (red shift) with increasing r_0. This redshift occurs because the energy difference E_{21} between the Ψ_2 and Ψ_1 states will decrease with increasing r_0. Moreover, it can be easily observed that with increasing r_0, the peak value of DCS decreases to a minimum and then increases rapidly. The physical origin is that the matrix element M is not a monotonic function of the ring radius r_0. Hence, the largest peak value and its position of DCS dependent strongly on the geometrical size of QRs.

In order to investigate the influence of the confinement strength $\hbar\omega_0$ on the Raman scattering, we set $r_0 = 5.0$ nm and plot, in Fig. 2, the DCS as a function of the diffusion photon energy for three different values, i.e., $\hbar\omega_0$=5.0, 8.0 and 10.0 meV, respectively, in the absence of external magnetic field. From Fig. 2, we find that the