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When physics meets signal processing: Image and video denoising based on Ising theory

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ABSTRACT

In this work we suggest a novel model for automatic noise estimation and image denoising. In particular, we investigate the useful affinity which arises between statistical mechanics and image processing, and describe a framework from which novel denoising algorithms can be derived: Ising-like models and simulated annealing techniques. This is the first time such algorithms are used for colored images and video denoising. Results, as well as benchmarks, suggest a significant gain in PSNR and SSIM in comparison to other filters, mainly in cases of low impulse noise. When hybridizing our models with other image processing techniques they are shown to be even more effective. Their major disadvantages- high complexity and limited applicability, are also discussed.

1. Introduction

Newton laws enable the prediction of the time evolution of a classical physical system given the initial conditions of its particles. However, when trying to describe macroscopic physical systems, with a typical number of 10^{23} particles, it is unpractical to solve all the equations of motion and find an analytical solution due to the computational burden involved. The best one can do is to find a statistical model describing the global characteristics of the system. This is why the tool-kit of statistical physics has become so important in modeling and solving problems from solid state and plasma physics to chemical and biological systems [1–3]. By modeling correctly the typical interaction between particles in the system, one can easily find the conditions for the equilibrium and its characteristic macroscopic parameters such as energy, temperature and pressure. In this paper, we discuss the similarities between these problems and the ones encountered when trying to extract as many information as possible regarding an image or a video signal.

In these cases as well, one tries to describe a large and complex system with specific correlations between its parts (pixels and/or frames). Additionally, in this field, there are cases where we have only partial knowledge about the pixels' values, hence there is a need to adopt probabilistic methods and estimations.

Utilizing this similarity, we employ our physical intuition regarding many-particles systems to the field of image processing and suggest a model for denoising of images and video signals degraded by additive external noise. Although the results are better than those of several known filters (Median, BM3D, Non-Local Means), we conclude that physical theory does not apply in a straightforward manner, as images do not always tend to minimal energy or maximal entropy similar to a physical system. Therefore, we suggest in this manuscript another improved model where Ising-like models are used together with basic image processing techniques, in order to achieve better image denoising.

The main contributions of this work are: employing for the first time an Ising-like model for denoising of colored images and video signals based on the L_1 norm, utilizing

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Perona and Malik diffusion coefficient, histogram analysis and other methods for automatic choice of the model's parameters and in some cases achieving better results than other commonly used filters. We emphasize that while Bayesian and statistical physics-based restoration techniques for gray-scale image denoising were suggested in previous works (for example see, [4–8]), to the best of our knowledge, Ising-like models were not used before to restore colored images and video signals. Such a regime requires new tools and techniques to cope with the layered and multi-dimensional nature of the data. Particularly, the 3D Ising-like model and the L_1 norm we used whose effectiveness has not been clarified yet in the research area of Markov random field. Furthermore, in order to improve past results, a better model for choosing the algorithm's parameters was needed.

The outline of the paper is as follows. In Section 2, we stress the analogy between physical systems and images and define the "thermodynamic" parameters characterizing an image. Sections 3 and 4 elaborate on Ising-like models and our proposed model respectively. In Sections 5 and 6 we focus on our recent algorithmic improvements where the latter section includes also automatic choice of parameters. Section 7 describes the theory and methods used to perform the simulation tasks given in this paper, while Section 8 presents the results. Section 9 summarizes the main contributions of the work and outlines future directions.

The paper is organized in a modular way, where in each section we try advance a step further towards efficient denoising of colored images.

2. Images as physical systems

Physical systems of many particles are described by average thermodynamic potentials [1]. The most basic of them is the internal energy, U, which can be represented in its differential form by

$$dU = TdS - pdV + \sum_i \mu i dNi + EdP + HdM \tag{1}$$

where T is the temperature, S is the entropy, p describes the pressure and V is the volume. μi is the chemical potential of type i particles and Ni is their number. E and H are the electric and magnetic fields, respectively. P is the polarization and M is the magnetization. Generally, the parameters above have some probabilistic distribution, yet in most applications only the average values are used.

From Eq. (1), one can deduce, for instance, the relation

$$T = \left(\frac{\partial U}{\partial S}\right)_{V,Ni,P,M} \tag{2}$$

which means that the temperature is the partial derivative of the energy with respect to the entropy when the volume, the number of particles, the polarization and the magnetization are fixed.

In an adiabatic system (where the entropy is constant), equilibrium is achieved by the condition of minimal energy [2]

$$dU = 0, d^2U > 0 \tag{3}$$

The analogy between thermodynamical systems and images is as follows: U, the internal energy of the image, can be assumed to be a global parameter which should be minimized in order to get an image of high quality. In our case, minimal energy means that images tend to be smooth, with adjacent pixels having similar values, but not too smooth. Deviations from this smoothness are maintained by randomness (thermal noise) and by some external knowledge we call an "external field". N, the number of particles in the physical system, can be used to describe the number of pixels in the noisy image. N determines the complexity of the algorithm. V is the area of the image, usually strictly relate to N. S, the physical (Boltzmann) entropy, is also well-defined for images in terms of Shannon entropy. p denotes the energy density, and T describes the energy required to reduce the entropy of the image. P and M can be thought of as external parameters affecting the image, e.g., distortions and noises where E and H denote the energy change they cause.

Using this analogy, we claim that physical models can be applied to the problem of describing images. In particular, two of these models are the Ising and Potts models which will be discussed in the next section. Since these models explicitly discuss phase transitions from disordered (which means noisy in the case of images and videos) to ordered phases (denoised images/video) we found them very useful for the image and signal processing tasks at hand. Indeed, being a probabilistic metaheuristic optimization technique, our model can be also related to the simulated annealing family [9] which serves as a bridge between physics and engineering. This viewpoint will be further discussed in Section 7.

3. The Ising model

A well-known model in the field of solid-state physics is the Ising model [3], which assumes a simple two-body interaction in a lattice of many spin-1/2 particles. Spin 1/2 is an internal degree of freedom which can take one of the values 1 or -1.

The typical energy of the system in the one dimensional Ising model is defined as follows:

$$E = -\frac{1}{2}\sum_{\langle i,j \rangle} J_{ij} S_i S_j - \sum_i h_i S_i. \tag{4}$$

where S_i and S_j denote the spins in the *i-th and, j-th* sites, J_{ij}describes the interaction between them, and h_i is the external magnetic field felt by the *i*-th site. The brackets indicate that i and j index neighboring positions and the 1/2 factor cancels double summation. $J_{ij} > 0$ models ferromagnetic substance, i.e., spins tend to align in the same direction (and thus reduce the interaction energy), while $J_{ij} < 0$ models anti-ferromagnets. This interaction term fits our intuition that most images are piecewise smooth. The generalization of the model to higher dimensions is straightforward: in the two/three dimensional models each spin has 4/6 nearest neighbors respectively. There exist more complex models, such as the one we eventually use, in which more neighbors are taken into account.