Chemical Physics 368 (2010) 93–99

Contents lists available at ScienceDirect

Chemical Physics

journal homepage: www.elsevier.com/locate/chemphys

Property of quantum tunneling in a driven triple-well potential

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ARTICLE INFO

Article history:
Received 23 July 2009
In final form 30 December 2009
Available online 6 January 2010

Keywords:
Triple-well potential
Quantum tunneling
Floquet formalism

ABSTRACT

The influence of parameters such as strength and frequency of a periodic driving force on the quantum tunneling of a particle in a triple-well potential is investigated, from the viewpoint of dynamical symmetry and the structure of the relevant Hilbert subspace both analytically and numerically. The numerical results show that, for different values of the parameters and locations of the particle, different kinds of tunneling are resulted. Tunneling occurs in special wells and could be suppressed completely or enhanced considerably, depending on different symmetries and structures of the Hilbert space where the tunneling dynamics takes place. The results are also analyzed within two- and three-state models in the light of Floquet formalism.

1. Introduction

The control of quantum tunneling by external driving force is an important subject in many areas of physics [1,2]. Depending on the strength and frequency of driving force, either suppression or enhancement of tunneling can be achieved. Lin and Ballentine found that tunneling enhancement could be obtained in a driven double-well potential, for relatively high strength and driving frequency close to the harmonic frequency at the bottom of each well. This effect is often called chaos-assisted tunneling (CAT) [3–5]. Classical mechanical analysis of the motion shows that the structure in classical phase space determines its tunneling behavior. Hanggi and co-workers discovered an opposite situation that for certain parameter ratios between strength and frequency, tunneling could be suppressed completely, termed coherent destruction of tunneling (CDT) [6–9]. In the latter case, the value of strength is rather small compared to that required in Lin's case. Observations of CAT and CDT in experiments have been reported [4,5,7,8]. Nonlinear CDT was reported in Bose–Einstein condensates [10], where suppression of tunneling occurred for a wide range of the parameter ratio. Moreover, classical nonlinear resonances in near-integrable phase space have been found to play a dominant role in tunneling, dubbed as resonance-assisted tunneling (RAT) [11]. Studies reveal that the tunnel splitting fluctuation is more intense in RAT than that in CAT, due to crossing of the regular tunnel doublets with a regular third state.

For about two decades quantum tunneling in double-well system and other physics system has been studied widely [1–11]. Recently, quantum tunneling and transport in triple-well potential have attracted a substantial attention [12–17], which has been practically realized and applied in chemical physics, atomic physics, and Bose–Einstein condensates. Maji et al. demonstrated that the tunneling period in a triple-well potential could be reduced or enhanced by coupling various perturbations [12]. Coherent tunneling by adiabatic passage in a triple-well structure potential was investigated very recently [13–15], which was suggested as a means to transport electrons from one well to another via adiabatic manipulation of the ground state. Many issues such as Josephson oscillation and Landau–Zener tunneling in Bose–Einstein condensates were discussed in a triple-well potential as well [16,17].

In the present paper, we shall conduct a numerical study on the quantum tunneling in a time-dependent driven triple-well potential. By adjusting the strength and frequency of driving force, various kinds of tunneling are achieved. The results show that unperturbed tunneling and non-resonant tunneling display a regular monochromatic oscillation between the two outer wells. The resonant tunneling presents a quick and complex pattern. For special parameter values of the driving force, tunneling could be suppressed completely and lead to CDT. In CAT case, tunneling appears in inner well and its rate is enhanced greatly. These characters are discussed within two- and three-state models based on analysis of the symmetry and the relevant Hilbert subspace in the light of Floquet formalism.

The organization of this paper is outlined as follow: Section 2 describes the driven triple-well potential and its feature. In Section 3, we present numerical results on the transition probability for initial wave packet localized in the outer or inner well. Section 4 is an analysis of the results based on two- and three-state models. Summary and conclusion are given in Section 5.

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doi:10.1016/j.chemphys.2009.12.029

2. The triple-well potential

The Hamiltonian of the periodically driven triple-well system under investigation has the form:

$$H(x,t) = H_0(x) + F(x,t), \tag{1}$$

and $H_0(x) = \frac{1}{2}p^2 + \frac{1}{2}\omega_0^2 x^2 (x^2 - a^2)^2$, $F(x,t) = Sx\sin(\omega t)$, where S and ω are strength and frequency of the external force, respectively. The unperturbed symmetric triple-well system $H_0(x)$ has three equally deep wells (left, central, and right) located at $x = -a$, 0, a, and two potential barriers located at $x = -a/\sqrt{3}$, $a/\sqrt{3}$, respectively. The profile of this unperturbed potential is illustrated in Fig. 1. Harmonic frequency of the inner well ($a^2\omega_0$) is different from that of the outer wells ($2a^2\omega_0$). The ground state of the triple-well system is approximately the ground state of harmonic oscillator localized at the inner well with energy approximately equal to $a^2\omega_0/2$. The first and second excited states are linear combinations of the ground states of harmonic oscillators localized at the outer wells with energies approximately equal to $a^2\omega_0$ (as shown in Fig. 1). For the system with above level structure, some remarkable characters of quantum tunneling are expected.

In analogy with double-well system [1], the invariance of unperturbed system $H_0(x)$ under parity operation $P : x \to -x$, $t \to t$ is destroyed by the driving force $F(x,t)$. However, a special dynamical symmetry of the system $H(x,t)$ remains, which is defined by the operation $P_T : x \to -x$, $t \to t + T/2$, where $T = 2\pi/\omega$. According to the Floquet theorem [1,18], there exists solution to this time-dependent Schrodinger equation that has the form: $\psi_n(x,t) = e^{-i\epsilon_n t}\phi_n(x,t)$. $\phi_n(x,t)$ is Floquet state obeying $\phi_n(x,t) = \phi_n(x, t+T)$, and ϵ_n is its quasienergy being a function of S and ω. The Floquet states may be classified into two kinds of states with even and odd parities with respect to the special discrete symmetry. As the parameters S and ω vary, quasienergies from different classes of symmetry may intersect, while quasienergies with the same class of symmetry will form avoided crossings.

In the concrete, the parameters are set as $\omega_0 = 1.5$, $a = 2.0$. Several lowest energy levels E_n and eigenfunctions $\varphi_n(x)$ of the unperturbed system $H_0(x)$ are presented in Fig. 1. The ground state $\varphi_0(x)$ located at the inner well has even parity. While the first and second excited states $\varphi_1(x)$, $\varphi_2(x)$ located at the outer wells have odd and even parities, respectively, and they are close but not degenerate in energy. In general, the eigenfunctions $\varphi_{2n}(x)$ of E_{2n} have even parity, while the eigenfunctions $\varphi_{2n+1}(x)$ of E_{2n+1} possess odd parity (n is integer). Then, we assume a Gaussian wave packet as the initial state: $\psi(x, t=0) = N\exp\left(-\frac{(x+a)^2}{2\sigma}\right)$, located at the bottom of the left well (see Fig. 1). N and σ are normalization factor and spread width of the initial wave packet, respectively. It is worth to note that $\psi(x,t=0)$ is approximately equal to the combination of $\varphi_1(x)$ and $\varphi_2(x)$: $\psi(x,t=0) \approx (\varphi_1(x) + \varphi_2(x))/\sqrt{2}$. For the solution $\psi(x,t)$ of the time-dependent Schrodinger equation, we use the second-order symplectic integrator [19]:

$$\psi(t+\Delta t) = \exp\left(-\frac{i}{\hbar}\frac{\Delta t}{2}V(t+\Delta t)\right)\exp\left(-\frac{i}{\hbar}\Delta t T\right) \times \exp\left(-\frac{i}{\hbar}\frac{\Delta t}{2}V(t)\right)\psi(t) + O(\Delta t^3), \tag{2}$$

where $T(x) = \frac{1}{2}p^2$ and $V(x,t) = \frac{1}{2}\omega_0^2 x^2(x^2-a^2)^2 + Sx\sin(\omega t)$. The spatial range is $[-4.0, 4.0]$ with the spatial mesh width $\Delta x = 8.0/2^7 = 0.0625$, while the time step is $\Delta t = 0.01$. The occupation probabilities in the three wells (left, central, and right) are defined as: $P_L(t) = \int_{-\infty}^{-a/\sqrt{3}} |\psi(x,t)|^2 dx$, $P_C(t) = \int_{-a/\sqrt{3}}^{a/\sqrt{3}} |\psi(x,t)|^2 dx$, and $P_R(t) = \int_{a/\sqrt{3}}^{\infty} |\psi(x,t)|^2 dx$, respectively, which give the time-varying probabilities of finding the wave packet in each well.

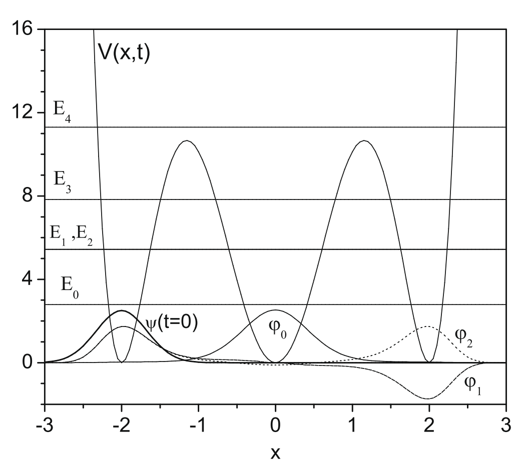

Fig. 1. The sketch of triple-well potential. The horizontal lines denote five lowest energy levels of the unperturbed system: $E_0 = 2.7874$, $E_1 = 5.4290$, $E_2 = 5.4408$, $E_3 = 7.8343$, and $E_4 = 11.3022$. $\varphi_0(x)$, $\varphi_1(x)$, and $\varphi_2(x)$ are eigenfunctions of E_0, E_1, and E_2, respectively. The initial state $\psi(x,t=0)$ is located at the bottom of the left well.

3. Numerical results

In this section, the effect of external strength S and frequency ω on occupation probabilities of the quantum particle shall be studied numerically. By tuning the values of parameters (S,ω) and the location of particle, we get non-resonant tunneling, resonant tunneling, CDT, as well as CAT in the triple-well potential.

3.1. Resonant tunneling

Let the initial state $\psi(x,t=0)$ located in the left well evolve, it may get spread and delocalized into the central and right wells. Fig. 2(a) shows the occupation probabilities $P_L(t)$, $P_C(t)$, and $P_R(t)$ as a function of time in the unperturbed case ($S = 0$). One may find that the wave packet just tunnels between the outer wells, similar to Rabi oscillation. There is hardly any probability in the inner well, indicating that it does not occur in the inner well. Energy remains conserved and below the top of barrier during the tunneling. The tunneling physics should be addressed as follows: the tunneling here is controlled by the reflection symmetry of this unperturbed system, and the doublet states φ_1 and φ_2 can describe the tunneling behavior completely.

Fig. 2(b) and (c) shows the occupation probabilities for various parameter values (S,ω), including non-resonant tunneling and resonant tunneling. In Fig. 2(b), at the non-resonant frequency $\omega = 1.5$, which could not induce resonance among energy levels, the third state is not involved effectively. Tunneling just occurs between the outer wells, displaying a regular sine-oscillation, the same as unperturbed tunneling. Here, the wave packet shows a coherent tunneling because of interference between the doublet states. When frequency value approaches to the intrinsic frequency $(E_n - E_m)$, tunneling displays some unexpected features. At $\omega = 2.4$ approximately equal to the fundamental resonance $(E_3 - E_1)$ as shown in Fig. 2(c), the excited state ϕ_3 is involved. The tunneling rate increases considerably and apparently displays chaotic oscillation. Remarkably, the resonant tunneling arises in the central well and presents a regular oscillation. However, in the double-well potential, fundamental resonance is $(E_3 - E_2)$ instead [1,6]. In the non-resonant case, time evolution of the wave packet mainly depends on the Floquet doublet states ϕ_1 and ϕ_2, both located in the outer wells, i.e., mainly in the Hilbert subspace spanned by (ϕ_1, ϕ_2). Therefore, tunneling presents a regular oscillation and only occurs in the outer wells. In the resonant case, ϕ_1 will transit to ϕ_3 with the same parity in short time, and the evolution of wave