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Dynamics of the quantum Duffing oscillator in the driving induced bistable regime ☆

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Abstract

We investigate the non-linear response of an anharmonic monostable quantum mechanical resonator to strong external periodic driving. The driving thereby induces an effective bistability in which resonant tunneling can be identified. Within the framework of a Floquet analysis, an effective Floquet–Born–Markovian master equation with time-independent coefficients can be established which can be solved straightforwardly. Various effects including resonant tunneling and multi-photon transitions will be described. Our model finds applications in nano-electromechanical devices such as vibrating suspended nano-wires as well as in non-destructive read-out procedures for superconducting quantum bits involving the non-linear response of the read-out SQUID.

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1. Introduction

Classical non-linear systems subjected to strong periodic external driving often have several stable stationary states for which the amplitudes and phases of the forced vibrations differ in size [1–3]. One of the simplest theoretical models which show the coexistence of two stable states induced by external driving is the well-known classical Duffing oscillator. An anharmonic statically monostable potential can be driven into a dynamically bistable regime showing various interesting features of non-linear response [1–3], such as hysteresis, period doubling, and thermal activation when finite temperatures are considered. The external driving field with frequency ω induces an effective dynamic bistability which is manifest by the non-monotoneous dependence of the amplitude A of the stationary vibrations for varying ω. For the classical system where all potential energies are allowed, this response curve $A(\omega)$ is smooth showing only two points of bifurcation for the related bistabilty. If the control parameter ω is additionally varied adiabatically, hysteretical jumps between the two stable states occur. If additional thermal noise is added to the system, the regime of bistabilty shrinks due to thermal escape of the metastable state.

The main subject of this work is to investigate the corresponding driven *quantum mechanical* system. The presence of time-dependent driving typically adds several interesting features to the properties of the time-independent quantum system, see for instance [4,5]. In this work, the focus is laid on the non-linear response of the driven quantum mechanical anharmonic oscillator in the presence of an Ohmic heat bath. We show that the non-linear response curve $A(\omega)$ displays beyond its characteristic shape additional quantum mechanical

☆ This work is dedicated to Prof. Phil Pechukas on the occasion of his retirement.

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resonances which are related to multi-photon absorptions. Additionally, we show that there exists a separation of time-scales indicating that the stationary state is reached by quantum tunneling from the dynamically induced metastable to the globally stable state. In fact, by tuning the control parameter ω, several resonant tunneling transitions can be identified as resonances in the corresponding tunneling rate.

The paper is organized in the following way: In Section 2, the system Hamiltonian is introduced. The latter is periodic in time, which allows the application of Floquet theory. Since we are interested in the stationary state in presence of (weak) dissipation, we introduce also a set of harmonic oscillators representing an Ohmic heat bath. An efficient way to determine the dynamics of the system is the use of a Born–Markovian master equation in the Floquet picture (Section 3) yielding a simple master equation with time-independent rate coefficients. After straight forward digitalization, the stationary oscillation amplitude A and the phase φ follow. In Sections 4 and 5, the non-linear response of A and φ depending on the various parameters is studied in detail. In Section 6, resonant tunneling is investigated. Finally, in Section 7, we discuss the applicability of our model to experimental systems before we conclude.

2. The quantum Duffing oscillator

The Hamiltonian for the driven anharmonic oscillator has the form

$$H_S(t) = \frac{p^2}{2m} + \frac{m\omega_0^2}{2}x^2 + \frac{\alpha}{4}x^4 + xf\cos(\omega t). \quad (1)$$

Here, m and ω_0 are the mass and the harmonic frequency of the resonator, respectively, while α gives the strength of the non-linearity. We focus on the case $\alpha > 0$ of hard non-linearities, where the undriven potential is monostable. The external driving is characterized by the amplitude f and the frequency ω. As it will become clear below, the driving induces an effective bistability in which quantum tunneling can be identified.

We include the effect of the environment by a bath of harmonic oscillators which are bilinearly coupled to the system with the coupling constants c_j [6]. The Hamiltonian for the bath and the coupling to the system is given by its standard form

$$H_B = \frac{1}{2}\sum_j \frac{p_j^2}{m_j} + m_j\omega_j^2\left(x_j - \frac{c_j}{m_j\omega_j^2}x\right)^2. \quad (2)$$

We focus on the generic case of an Ohmic bath with the spectral density

$$J(\omega) = \frac{\pi}{2}\sum_j \frac{c_j^2}{m_j\omega_j}\delta(\omega - \omega_j) = m\gamma\omega e^{-\omega/\omega_c} \quad (3)$$

with damping constant γ and cut-off frequency ω_c. The total Hamiltonian is $H(t) = H_S(t) + H_B$.

To proceed, we scale $H(t)$ to dimensionless quantities such that the energies are in units of $\hbar\omega_0$ while the lengths are scaled in units of $x_0 \equiv \sqrt{\frac{\hbar}{m\omega_0}}$. The non-linearity parameter α is scaled in units of $\alpha_0 \equiv \hbar\omega_0/x_0^4$, while the driving amplitude is given in units of $f_0 \equiv \hbar\omega_0/x_0$. Moreover, we scale temperature in units of $T_0 \equiv \hbar\omega_0/k_B$ while the damping strengths are measured with respect to ω_0.

To investigate the dynamical behavior of the driven resonator, it is convenient to use the periodicity of $H_S(t)$ with respect to time and switch to the Floquet picture [7], the later being equivalent to a transformation to the rotating frame. The Floquet or quasienergies ε_α follow from the solution of the eigenvalue equation

$$\left[H_S(t) - i\hbar\frac{\partial}{\partial t}\right]|\phi_\alpha(t)\rangle = \varepsilon_\alpha|\phi_\alpha(t)\rangle \quad (4)$$

with the Floquet states $|\phi_\alpha(t)\rangle$. The quasienergies ε_α are defined up to a multiple integer of $\hbar\omega$, i.e., the state $|\phi_\alpha^{(n)}(t)\rangle = e^{in\omega t}|\phi_\alpha(t)\rangle$ is also an eigenstate of the Floquet Hamiltonian, but with the eigenvalue $\varepsilon_{\alpha,n} = \varepsilon_\alpha + n\hbar\omega$. This feature prevents us from a global ordering of the quasienergies which, however, can be achieved with the mean energies obtained after averaging over one driving period, i.e.,

$$\overline{E}_\alpha = \sum_n (\varepsilon_\alpha + n\hbar\omega)\langle c_{\alpha,n}|c_{\alpha,n}\rangle \quad (5)$$

with the Fourier components of the Floquet states [7]

$$|c_{\alpha,n}\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\, e^{in\omega t}|\phi_\alpha(t)\rangle. \quad (6)$$

3. Dynamics of the quantum Duffing oscillator

3.1. Floquet–Born–Markovian master equation

The dynamics of the resonator in the regime of weak coupling to the bath can be efficiently described by a Born–Markovian master equation in the Floquet picture [7] for the elements $\rho_{\alpha\beta}(t) \equiv \langle\phi_\alpha(t)|\rho|\phi_\beta(t)\rangle$ of the reduced density operator ρ after the harmonic bath has been traced out in the usual way. For weak damping, the dissipative influence of the bath is relevant only on a time scale much larger than the driving period $T_\omega = 2\pi/\omega$. Thus, the time-dependent coefficients which are periodic in time with period T_ω can safely be replaced by their average over one driving period (*moderate rotating wave approximation* [7]). This yields a simplified master equation with time-independent coefficients which reads

$$\dot{\rho}_{\alpha\beta}(t) = \sum_{\alpha'\beta'} \mathcal{M}_{\alpha\beta,\alpha'\beta'}\rho_{\alpha'\beta'}(t) \quad (7)$$