Available online at www.sciencedirect.com

SCIENCE DIRECT®

Signal Processing: *Image Communication* 20 (2005) 569–581

SIGNAL PROCESSING: IMAGE COMMUNICATION

www.elsevier.com/locate/image

Implications of invariance and uncertainty for local structure analysis filter sets

Hans Knutsson*, Mats Andersson

Medical Informatics, Department of Biomedical Engineering, Center for Medical Image Science and Visualization, Linköping University, University Hospital, SE-581 15 Linköping, Sweden

Received 14 March 2005

Abstract

The paper discusses which properties of filter sets used in local structure estimation that are the most important. Answers are provided via the introduction of a number of fundamental invariances. Mathematical formulations corresponding to the required invariances leads up to the introduction of a new class of filter sets termed *loglets*. Loglets are polar separable and have excellent uncertainty properties. The directional part uses a spherical harmonics basis. Using loglets it is shown how the concepts of quadrature and phase can be defined in *n*-dimensions. It is also shown how a reliable measure of the certainty of the estimate can be obtained by finding the deviation from the signal model manifold.

Local structure analysis algorithms are quite complex and involve a lot more than the filters used. This makes comparisons difficult to interpret from a filter point of view. To reduce the number 'free' parameters and target the filter design aspects a number of simple 2D experiments have been carried out. The evaluation supports the claim that loglets are preferable to other designs. In particular it is demonstrated that the loglet approach outperforms a Gaussian derivative approach in resolution and robustness.

Keywords: Generalized phase; Invariance; Uncertainty; Quadrature filters; Local structure tensor; Spherical harmonics; Orientation estimation; Velocity estimation

1. Introduction

The first steps towards analysis of images were taken 4 decades ago. From the very start detecting edges and lines in images was considered a fundamental operation [31,17]. Since these early days new and more advanced schemes for analysis

*Corresponding author. Tel.: +46 13227294; fax: +46 13101902.

E-mail address: knutte@imt.liu.se (H. Knutsson).

URL: http://www.imt.liu.se/mi.

0923-5965/$ - see front matter
doi:10.1016/j.image.2005.03.011

of local image structure has been suggested in a seemingly never ending stream Local image orientation, scale, frequency, phase, motion and locality of estimates are prominent examples of features that have been considered central in the analysis [12,13,24,27,6,5,28,36,19,29,20,9,32,21,2,26,34,35,22,15,8,4,11,33].

Apart from sheer curiosity, the main force driving the research has been the need to analyze data produced by increasingly capable imaging devices. Presently produced data are also often intrinsically more complex. Both the outer and the inner dimensionality can be higher, e.g. volume sequence data and tensor field data respectively.

Regardless of this development the first stages in the analysis remain the same. In most cases the processing starts by performing local linear combinations of image values, e.g. convolution operators. Perhaps somewhat surprising after 30 years of research the design of these filters is still debated. In fact the object of this paper is to contribute to this discussion in a way that hopefully will help in bringing it to an end by providing what we believe to be a valid line of reasoning.

1.1. Image signal models

The mathematical framework used to characterize image signals is the foundation for development and evaluation of all image processing methods. Methods in image processing can in most cases be classified as belonging to the deterministic or the statistical world. This is unfortunate since none of the approaches alone is well suited for modeling real life images. Edges, lines, corners etc are naturally deterministically modeled whereas textures and noise belong naturally to the statistical world. Real life images clearly has components from both worlds. Objects moving in front of textured backgrounds and borders between textured areas provide obvious examples. A fusion of appropriate parts from both worlds will potentially provide a much more powerful image analysis model. This is the spirit in which the remainder of this paper should be read.

2. What, exactly, is orientation and motion

There is a strong correspondence between the problems of estimating velocity and estimating signal orientation. If the signal is band-limited so as to not contain frequencies above the Nyquist limit the problems are in fact identical. For the case of constant illumination this identity is manifested in the Fourier domain by that all non-zero values can be found on a plane through the origin. The normal to the plane, $\hat{\boldsymbol{m}}$, is directly related to the velocity through:

$$\boldsymbol{v} = \frac{\boldsymbol{P}_x \hat{\boldsymbol{m}}}{\boldsymbol{P}_t \hat{\boldsymbol{m}}}, \qquad (1)$$

where $\boldsymbol{P}_x$ projects $\hat{\boldsymbol{m}}$ onto the spatial frequency plane, $\boldsymbol{P}_t$ projects $\hat{\boldsymbol{m}}$ onto the temporal frequency axis.

2.1. Invariances and images of reality

In a Newtonian world the true motion and orientation of a rigid object is a well defined entity that is obviously independent of the visual appearance of the object itself. When orientation and/or velocity is estimated using images it is, however, equally obvious that the properties of, for example, the imaging device, the light sources and the object surface directly influence the transfer of pertinent information, see e.g. [16]. For this reason, a fundamental part of any estimation method is the incorporation of appropriate invariances. The implications of a number of important invariances are discussed below.

2.2. Event identity and estimate locality

For a single highly localized feature it is, all else being equal, desirable to maximize spatial locality of the feature estimate. Retaining feature identity also requires that the estimate is smoothly varying and centered on the feature.[1] These two requirements counteract each other in a fundamental way and there exist many possibilities to define

[1] It would be reasonable to require a unimodal response but we will not do so as it may be perceived as giving the quadrature approach to much of an advantage.