Chemical Physics Letters 660 (2016) 283–286

Contents lists available at ScienceDirect

Chemical Physics Letters

journal homepage: www.elsevier.com/locate/cplett

Research paper

Electrically forced unpinning of spiral waves from circular and rectangular obstacles

Porramain Porjai [a,b], Malee Sutthiopad [a], Jiraporn Luengviriya [c,d], Metinee Phantu [a], Stefan C. Müller [e], Chaiya Luengviriya [a,*]

[a] Department of Physics, Kasetsart University, 50 Phaholyothin Road, Jatujak, Bangkok 10900, Thailand
[b] Faculty of Science and Technology, Valaya Alongkorn Rajabhat University Under the Royal Patronage, 1 Moo 20 Phaholyothin Road, Khlong Neung, Klong Luang, Phathum Thani 13180, Thailand
[c] Department of Industrial Physics and Medical Instrumentation, King Mongkut's University of Technology North Bangkok, 1518 Pibulsongkram Road, Bangkok 10800, Thailand
[d] Lasers and Optics Research Group, King Mongkut's University of Technology North Bangkok, 1518 Pibulsongkram Road, Bangkok 10800, Thailand
[e] Institute of Experimental Physics, Otto-von-Guericke University Magdeburg, Universitätsplatz 2, D-39106 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 24 June 2016
In final form 19 August 2016
Available online 21 August 2016

ABSTRACT

Unpinning of spiral waves from circular and rectangular obstacles of various sizes under electrical forcing was investigated in both the Belousov-Zhabotinsky reaction and numerical simulations with the Oregonator model. The used rectangles had a similar area size but different circumferences by adjusting width and height, while the area of the circles increased with the circumference. The results showed that for both cases the necessary strength of forcing for unpinning increased with the circumference. This implies that for such unpinning the circumference has a higher impact than the area, at least for circular and rectangular obstacles.

1. Introduction

Spiral waves can be observed in different excitable media such as CO oxidation on platinum surfaces [1], cell aggregation in slime mold colonies [2], electrical wave propagation in cardiac tissues [3], and the Belousov-Zhabotinsky (BZ) reaction [4,5]. It is known that electrical spiral waves on the heart are associated with cardiac tachycardia and life-threatening fibrillations [6,7]. Spiral waves are annihilated when their tips drift and hit the boundary of the medium. However, their lifetime is enhanced, if they are pinned to anatomical inhomogeneities or obstacles, e.g., veins or scars [3].

A high-voltage electrical shock can erase simultaneously all propagating waves in the cardiac system but it potentially causes tissue damages so that alternative treatments using a much lower voltage have been being investigated. An application of low-amplitude electric field from a pair of electrodes, known as the virtual electrode method, can modulate the membrane potential near obstacles and induce additional electrical waves [8–11]. The amplitude of electric field for inducing the waves depends on the orientation [9,10] and the curvature [11] of the obstacles.

Other low-voltage method uses a high-frequency train of local stimuli to overcome spiral waves in cardiac tissue cultures [12–16]. In the BZ reaction, a high-frequency wave train can be produced by a droplet of high concentrated H_2SO_4 since the acid increases the local excitability and ignite waves. The unpinning of spiral waves from circular obstacles by the wave train has been demonstrated [15,16] for different obstacle types, i.e., using oil droplets and laser spots as hard and soft obstacles, respectively. The necessary frequency of the wave train for unpinning increases with the radius of circular obstacles [12–16]. The unpinning may fail, if the spiral waves are pinned to very large obstacles. The success of unpinning can be improved by decreasing the excitability of the medium because the free spiral core is enlarged [14].

Electrically forced unpinning of spiral waves in the BZ reaction has been investigated in different situations. Unlike the virtual method in cardiology, the applied electric field does not induce wave emissions from obstacles but it causes an advective motion of ionic species. In the absence of obstacles, the electric field induces a drift of freely rotating spiral waves [17–19] as well as a reorientation of scroll rings (three-dimensional spiral waves with ring-shaped filaments) [20,21]. Under the forcing, spiral waves are detached from circular obstacles, when the density of the applied electrical current reaches a critical value which increases with the obstacle diameter [22]. For a given circular obstacle, this current density increases with the concentration of H_2SO_4, which determines the medium excitability and the core size of free spiral waves – the higher $[H_2SO_4]$, the higher the excitability and the

---

[*] Corresponding author.
E-mail address: fscicyl@ku.ac.th (C. Luengviriya).

http://dx.doi.org/10.1016/j.cplett.2016.08.052
0009-2614/

smaller the spiral core [23]. These facts imply that it is more difficult to release spiral waves from larger circular obstacles. For three-dimensional BZ media, an applied electric field can release scroll rings from spherical plastic beads [24].

Recently, we have reported the properties of spiral waves pinned to circular and rectangular obstacles [25]. While they are pinned to obstacles with different shapes, the spiral waves have common features: their wave period, wavelength and wave velocity increase with the circumference of the obstacle, regardless of its area. In this article, we present a further study on the electrically forced release of spiral waves from circular and rectangular obstacles, both in experiments with BZ media and in numerical simulations using the Oregonator model [26,27].

2. Experiments

We prepared the Belousov-Zhabotinsky (BZ) solutions as described in previous report [25]. The initial concentrations were as follows: $[H_2SO_4] = 160$ mM, [MA] = 50 mM, $[NaBrO_3] = 50$ mM, and [ferroin] = 0.625 mM. The BZ reaction was embedded in a 1.0% w/w agarose gel to prevent any hydrodynamic perturbation. Unpinning of spiral waves by electrical forcing was investigated in a flat reactor (volume $100 \times 100 \times 1.0\ \mathrm{mm}^3$) with left and right boundaries attached to electrolytic compartments (size of each $25 \times 100 \times 2.0\ \mathrm{mm}^3$). Eight circles with different diameters of 1.5, 1.9, 2.5, 2.8, 3.1, 3.5, 3.9, and 4.5 mm (areas = 1.8–15.9 mm^2 and circumferences = 4.7–14.1 mm) and four rectangles with a similar size of area (6 mm^2) and different dimensions of 2.3×2.6, 4.6×1.3, 4.9×1.2, and $6.5 \times 0.9\ \mathrm{mm}^2$ (circumferences = 9.8–14.8 mm) were used as obstacles.

A spiral wave pinned to an obstacle was created by using a two-layer method (cf. Fig. 1 in [22]). A volume of the BZ solution was poured into the reactor as the first layer of about 2.5 cm height, where the obstacle was prior set. A silver wire of 0.5 mm diameter was immersed for a few seconds into the medium to reduce the local concentration of the inhibitor Br^- so that a wave front was ignited. When one open end of the wave front approached the obstacle, another volume of the solution was added as the second layer leading to the final height of 4.5–5.0 cm. Then the wave front curled in and formed a spiral wave pinned to the obstacle.

After a transient interval of a few rotations, i.e., when the form of the spiral wave was stable, an electrical current was applied through electrodes in the electrolytic compartments which connected series with the main volume. As a result of the electrical current, gas bubbles were formed near the surface of the electrodes and caused a temporal fluctuation of the resistance in the system. Therefore, a power supply with a constant electrical current mode was used and the electrical current density was reported in our experiments instead of the electric field, usually used in simulations.

Starting from a low density, the constant electrical current was applied to the reaction for a few spiral rotations before it was increased by a step of $\Delta J = 2\ \mathrm{mA\,cm}^{-2}$, until the spiral wave was detached from the obstacle. The reactor was set vertically in a transparent thermostated water bath to control the temperature of the medium at 15 ± 1 °C. The bath was put between a white light source and a color CCD camera (Super-HAD, Sony) to record the images of the medium every second with a resolution of 0.1 mm pixel^{-1}.

Fig. 1 illustrates the unpinning of spiral waves from obstacles having a similar area of about 6 mm^2. In Fig. 1(a), the spiral wave was released from the circular obstacle, when the density of the electrical current reached a critical value $J_{\mathrm{unpin}} = 57\ \mathrm{mA\,cm}^{-2}$. When the obstacle was changed to a rectangle of 2.3 mm × 2.6 mm in Fig. 1(b), the spiral wave was unpinned at a higher critical value $J_{\mathrm{unpin}} = 67\ \mathrm{mA\,cm}^{-2}$. Even more, as shown in Fig. 1(c), the unpinning for a longer rectangle of 6.5 mm × 0.9 mm occurred at a much higher critical value $J_{\mathrm{unpin}} = 160\ \mathrm{mA\,cm}^{-2}$.

Fig. 2 shows the critical current density J_{unpin} found in experiments using circular and rectangular obstacles. As in Fig. 2(a), J_{unpin} increased with the area A of the circular obstacles. Even though they have a very similar area size, the rectangular obstacles with different width and height resulted also in different J_{unpin} values. The plot in Fig. 2(b) illustrates the relation between J_{unpin} and the circumference l. For all obstacles, J_{unpin} always increased with the circumference l, while the area A was either increased (for circles) or remained constant (for rectangles).

3. Simulations

In our numerical simulations, the two-variable Oregonator model was used to describe the dynamics of the activator u and the inhibitor v, corresponding to the concentrations of $HBrO_2$ and the catalyst in the BZ reaction, respectively. The advection terms for both u and v account for the electric field E applied in x-direction

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon}\left(u - u^2 - f v \frac{u-q}{u+q}\right) + D_u \nabla^2 u - M_u E \frac{\partial u}{\partial x},$$
$$\frac{\partial v}{\partial t} = u - v + D_v \nabla^2 v - M_v E \frac{\partial v}{\partial x}. \qquad (1)$$

The parameter values were chosen as $\varepsilon = 0.01$, $q = 0.002$, $f = 1.4$, the diffusion coefficients $D_u = 1.0$ and $D_v = 0.6$, and the ionic mobilities $M_u = -1.0$ and $M_v = 2.0$. We used an explicit Euler method with a nine-point approximation of the two-dimensional Laplacian operator and a centered-space approximation of the gradient term with a uniform grid space $\Delta x = 0.1$ s.u. and a time step $\Delta t = 3.0 \times 10^{-3}$ t.u., as required for numerical stability ($\Delta t \leqslant (3/8)\,\Delta x^2$ [28]).

We investigated five series of obstacles. The first two series are circles and rectangles which follow the experiments: (I) nine circles with diameters of 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, and 10.0 s.u. (areas = 3.1–78.6 $\mathrm{s.u.}^2$ and circumferences = 6.3–31.4 s.u.) and (II) six rectangles with a fixed size of area (1.70 $\mathrm{s.u.}^2$) and different dimensions of 1.3×1.3, 1.7 × 1.0, 3.4 × 0.5, 5.4 × 0.3, 8.5 × 0.2 and 17.0 × 0.1 $\mathrm{s.u.}^2$ (circumferences = 5.2–34.2 s.u.). Similar to the circles, the third series are squares whose area and circumference simultaneously increase with the side length: (III) five squares with dimensions 3.0 × 3.0, 4.5 × 4.5, 6.0 × 6.0, 7.5 × 7.5, and 9.0 × 9.0 $\mathrm{s.u.}^2$ (areas = 9.0–81.0 $\mathrm{s.u.}^2$ and circumferences = 12.0–36.0 s.u.).

To elaborate more the effect of the obstacle area, the two last series of rectangles and ellipses were utilized: (IV) six rectangles with a fixed circumference (34 s.u.) and different dimensions of 17.0 × 0.1, 16.1 × 1.0, 15.1 × 2.0, 13.1 × 4.0, 11.1 × 6.0, and 8.6 × 8.5 $\mathrm{s.u.}^2$ (areas = 1.7–73.1 $\mathrm{s.u.}^2$), and (V) five ellipses with a fixed circumference (34 s.u.) and different dimensions of 8.5 × 0.56, 8.25 × 1.4, 8.0 × 1.98, 7.5 × 2.89, and 7.0 × 3.64 $\mathrm{s.u.}^2$ (areas = 15.0–80.1 $\mathrm{s.u.}^2$).

For each simulation, one unexcitable obstacle was defined in the middle of the system and both boundaries of the obstacle and the system had no-flux conditions described in detail in [23]. A pinned spiral wave was initiated by stimulating a planar wave at an edge of the system. When the wave front reached the obstacle, half of the system was reset to an excitable state ($u = 0$ and $v = 0$) leading to a planar wave with one end traced the boundary of the obstacle. Subsequently, the wave front curled to form a spiral wave pinned to the obstacle. The spiral wave was allowed to propagate until its structure was stable. Similar to the experi-