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Research paper

Modes competition in superradiant emission from an inverted sub-wavelength thick slab of two-level atoms

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ABSTRACT

Using the expansion in the eigenmodes of 1-D Lienard-Wiechert kernel, the temporal and spectral profiles of the radiation emitted by a fully inverted collection of two-level atoms in a sub-wavelength slab geometry are computed. The initial number of amplifying modes determine the specific regime of radiation. In particular, the temporal profile of the field intensity is oscillatory and the spectral profile is non-Lorentzian with two unequal height peaks in a narrow band centered at the slab thickness value at which the real parts of the lowest order odd and even eigenvalues are equal.

1. Introduction

Following similar steps to those found in Feynman and Dyson's seminal work in quantum electrodynamics [1,2] we computed using field theoretical techniques, the atomic and photonic self-energies in an ensemble of dense homogeneously broadened two-level atoms [3,4]. The real part of the reducible atomic self-energy generalized the expression derived by Dicke for the superradiant lifetime of very small samples (as compared to the atomic transition wavelength) [5], to also samples with slab and spherical geometries having sizes equal or larger than the transition wavelength. The imaginary part of the reducible atomic self-energy, dubbed the Cooperative Lamb Shift because of it having similar origin as the isolated atom Lamb shift [6], except that in this instance, to lowest order of perturbation theory the photon is not emitted and absorbed by the same atom but by two different atoms of the same ensemble, was also computed for the slab and spherical geometries. The expression for the Cooperative Lamb Shift obtained in [3] for the slab correctly predicted the results observed in milestone experiments recently performed at DESY [7] and at Durham [8].

For weak fields and weak initial atomic excitation, the effect of including one-photon exchange is, inter alia, to modify the location of the pole in the complex plane of the spectral distribution of the transmission coefficient in a slab from its location in the expression of the dielectric constant.

In the nonlinear regime, while the reduced atomic self-energy still represents the rate of change of the power loss from the atomic system at initial time [9]; the spectral distribution of the superradiant emission from an inverted system can be obtained only if one knew the solutions of the coupled Maxwell-Bloch (MB) equations for the system.

However, the MB system of partial differential equations is non-linear and is in general not amenable to an analytical solution except under severe simplifying assumptions such as including only a single mode in the plane-wave expansion of the electromagnetic field, making the Rotating-Wave-Approximation, the Slowly Varying Envelope Approximation in Space in Maxwell equation, and/or neglecting the relaxation terms in both the diagonal and off-diagonal elements of the atomic density matrix of Bloch equations. All terms that are neglected under these different approximations are kept in the present calculations.

For the purpose of solving the MB system in 1-space, 1-time dimensions, it was found convenient to expand the dynamical variables in the complete basis formed by the eigenfunctions of the Lienard-Wiechert kernel [10]. This expansion allowed the reduction of the MB system of partial differential equations into an equivalent system of infinite system of coupled ordinary differential equations [11] which can be easily solved by standard computer routines. This mathematical reformulation of the problem also allowed a more efficient and accurate determination of the solutions of the MB and it further provided the tools for achieving better transparency in analyzing physically the obtained numerical solutions.

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In this paper, i use the above steps of reducing the MB system of partial differential equations to a system of infinite ordinary differential equations to compute for an initially inverted system the temporal and spectral profiles of the radiation emitted from a slab as function of the slab thickness, when this later quantity is smaller than the radiation wavelength. The interest in this regime is motivated by a desire to obtain an expression for the Cooperative Lamb Shift for an inverted system and by the recent investigations in nano-photonics, where it was observed that the emission line shapes are non-Lorentzian and have two unequal heights peaks [12] in experiments in dense and cold atomic system of ellipsoidal shapes.

The accuracy of the presented numerical calculations is determined by the number of modes that are incorporated in the truncated system of infinite coupled nonlinear ordinary differential equations. It is verified by ensuring that the results using a finite basis of n-eigenfunctions and those from $(n+1)$-eigenfunctions basis are everywhere equal to within an error of less than 1 part per million.

The present calculations on the spectral line-shape of the emission from an inverted system complements previous work by the author on the emission spectral distribution from a slab which was initially weakly excited and reported on in [13] and that shows for very thin samples two equal heights principal peaks.

The results of the inverted system exhibit new spectral features not present, as well, in other works in what is now commonly called the single photon superradiance regime [14–16]. In these works, the atomic population time-changing dynamics is not considered, which is a good approximation only in the linear regime, i.e. when initially only few atoms are excited.

The main results of this paper are that:

- The continuous model of the Maxwell-Bloch equations may produce an emission spectral shape with two unequal heights peaks in a narrow range of the slab thickness.
- The two unequal heights peaks are a result of the competition between the lowest order even and odd modes which occurs when the real parts of these modes eigenvalues are approximately equal.
- The two peaks frequencies are obtained from the dressed eigenvalues of the two normal branches of the nonlinearly interacting bare lowest order even and odd modes.
- The different sign of the frequency shifts in the position of the single peak observed on either side of the transition region is attributable to which of the two bare modes is leading. This difference in sign is due to the opposite signs in the imaginary parts of the eigenvalues of the lowest orders even and odd modes.

This paper is organized as follows: in Section 2, i write the Maxwell-Bloch equations in normalized coordinates suitable for the present problem; in Section 3, i give the results of the numerical computations for the temporal profile and the spectral profile of the emitted radiation for different slab thicknesses. I show also that the different observed emission regimes can be a priori identified through a comparison between the values of the real parts of the lowest order even and odd modes and a comparison between these values with the value of the normalized transverse atomic relaxation width. In the Appendix, the details of some mathematical expressions not incorporated in the text are given to facilitate the reading of the text.

2. Maxwell-Bloch equations in normalized coordinates

To facilitate the reading of the manuscript, i include in this section material which appeared previously but which omission was reported to be undesirable.

First, i give the standard form of the Maxwell-Bloch equations when written in normalized variables form, convenient for the present problem.

Defining the normalized variables for a slab of thickness $2z_0$, as:

$$Z = z/z_0,\ T = Ct,\quad \Gamma_1 = \gamma_1/C,\ \Gamma_2 = \gamma_2/C,\ \Gamma_{frgn} = \gamma_{frgn}/C,$$
$$u_0 = k_0 z_0,\ \Omega_{c,0,L} = \omega_{c,0,L}/C,$$

where $k_0 = \frac{2\pi}{\lambda_0}$, λ_0 is the wavelength corresponding to the transition frequency, $\Omega_{c,0,L}$ are respectively the normalized electric field carrier frequency, the atomic transition frequency, and the Lorentz shift. In this system of units, all quantities are normalized to the parameter of interatomic cooperativity $C = \frac{4\pi N \wp^2}{\hbar V}$, where N is the number of particles, V is the slab volume, and $\wp$ is the reduced dipole moment of the atomic transition (its normalization is uniquely determined when given as function of the isolated atom decay rate, see below). The relaxation decay rates γ_1, γ_2, refer respectively to the longitudinal decay rate, and the resonant transverse decay rates. In the normalized units, the transverse resonant decay rate Γ_2, due to the instantaneous dipole-dipole interaction between identical atoms, is equal to 2.33/4 for a $J = 0 \to J = 1$ transition, and the normalized Lorentz shift is equal to 1/3 [3]. The isolated atom decay rate $\gamma_1 = \frac{4}{3}\wp^2 k_0^3/\hbar$ specifies the longitudinal decay rate of the atomic system.

The Maxwell-Bloch equations in 1-D are given in these units, for $\zeta = C z_0/c \ll 1$, where c is the speed of light in vacuum, by

$$\frac{\partial \chi(Z,T)}{\partial T} = -[i(\Omega_0 - \Omega_c) + \Gamma_T - i\Omega_L n(Z,T)]\chi(Z,T) + \frac{i}{2} n(Z,T)\psi(Z,T), \tag{1}$$

$$\frac{\partial n(Z,T)}{\partial T} = -i[\chi^*(Z,T)\psi(Z,T) - \chi(Z,T)\psi^*(Z,T)] + \Gamma_1(1 - n(Z,T)), \tag{2}$$

$$\psi(Z,T) = iu_0 \int_{-1}^{1} dZ' \chi(Z',T) \exp\left(iu_0|Z - Z'|\right), \tag{3}$$

where $\Gamma_T = \Gamma_2 + \frac{\Gamma_1}{2}$, χ (complex) and n (real) describe respectively the atomic polarization density and the degree of excitation of the two-level atoms ($n = 1$ if all atoms are in the ground state and $n = -1$ if all atoms are excited), and ψ represents the normalized Rabi frequency of the complex electric field envelope. It should be noted that Γ_T is the width of the Lorentzian atomic polarization for low atomic excitation (linear theory).

The reader should note that in the above equations:

- The dynamical variables are functions of both space and time; i.e. the discrete index designating the different atoms in the ensemble has been replaced by a continuous spatial variable.
- The kernel of the integral equation, Eq. (3), is the one-dimensional form for the Lienard-Wiechert potential, shown in [3] to represent the effective interaction (one-photon exchange) between a ground state atom and an excited state atoms, and that was shown to be responsible for cooperative phenomena (i.e. superradiance and Cooperative Lamb Shift).

The system described by Eqs. (1)–(3) is solved by expanding each of the quantities $\psi(Z,T), n(Z,T), \chi(Z,T)$ in the basis formed by the eigenfunctions of the integral equation

$$\Lambda_s^{e,o}\phi_s^{e,o}(Z) = \frac{u_0}{2}\int_{-1}^{1} dZ' \exp\left(iu_0|Z - Z'|\right)\phi_s^{e,o}(Z') \tag{4}$$

where $\Lambda_s^{e,o}$ is the eigenvalue of the integral equation, and e and o in the superscript denote respectively the even and odd families of solutions and the subscript s specifies the order of the eigenfunction.