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Controlling mobility via rapidly oscillating time-periodic stimulus

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ABSTRACT

To address the dynamics of a Brownian particle on a periodic symmetric substrate under high-frequency periodic forcing with a vanishing time average, we construct an effective Langevin dynamics by invoking Kapitza–Landau time window. Our result is then exploited to simulate the mobility both for original and effective dynamics which are in good agreement with theoretical predictions. This close agreement and the enhancement of mobility are very robust against the tailoring of amplitude-to-frequency ratio which substantiates the correctness of our calculation. Present results may be illuminating for understanding the dynamics of cold atoms in electromagnetic fields.

1. Introduction

Theoretical study of the dynamics of a particle moving in a rapidly oscillating potential (ROP) is a recurrent theme in various contexts [1–15]. Well-known paradigms are the Paul trap [3] and electro- and magneto-dynamic traps [16], neutral atoms [17,18], billiard traps [19], and rapidly scanning optical tweezers [20]. Many interesting and curious experimental observations stem out of the interaction of the system with the ROPs. One can stabilize a state, which shows potential instabilities under the absence of external fields, by a simple application of a rapidly oscillating field. Such an exercise lends a good degree of control over a system and opens up possibilities to explore varied vistas of general physical curiosity. In order to gain some insights that are offered by systems modulated by rapidly oscillating potentials, a descriptive theory of their dynamics is not only indispensable but also useful [21]. Motivated by these facts, in the present Letter, we intend to investigate the response of mobility of a Brownian particle in the presence of ROP in conjunction with friction, one of the most important classes of nonequilibrium systems. The behavior of the studied system is much more versatile, and partly counterintuitive effects can occur. Well-studied effects are the absolute negative mobility [22], stochastic resonance [23], or the rectification of noise in ratchets and Brownian motors [24,25]. At this juncture, we want to mention that the invention of experimental techniques like optical tweezers [20] or the atomic force microscope [26] have made it possible to exert quickly varying perturbations like forces on nano-sized objects and thus to experimentally study the proposed effects. Recent years have witnessed intense activity in the studies of the transport phenomena in the presence of periodic perturbations [27–31]. Moreover, the Brownian dynamics in the presence of external perturbation is a very useful tool to model various challenging and interesting phenomena [32,33].

When the external perturbation varies sufficiently slowly with time, the system is essentially in equilibrium with the instantaneous potential. If that is not the case, the solution to the problem becomes a lot more difficult because no general methodology is available, as we have already mentioned. It is this regime that our work deals with. Here, we plan to investigate the mobility of a Brownian particle subjected to an external time-oscillatory drive of zero mean. We calculate the mobility in the parameter regime where the time scales of the dynamics of the system under study are much larger than the period of the driving itself. We focus on the result that this dynamics is equivalent to the one in which the periodic perturbation is replaced by a time independent effective potential. This important result has been proposed by Kapitza and later by Landau and Lifschitz within the framework of classical mechanics [1,2]. The method of Kapitza–Landau-Lifschitz (KLL) has been also applied to the stabilization of a matter-wave soliton in two-dimensional Bose–Einstein condensates without an external trap [34]. The theory of KLL is based on the separation of the particle's motion into a slow and a fast changing part. In their theory, it was observed that the particle's real motion is approximately identified by its slow part, such that the real motion can be

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characterized by replacing its time dependent equation of motion by the time independent equation of motion of the slow part. The fast motion results in an effective potential (that has a local minimum) for the slow motion. As a result, a particle can be trapped at a minimum of the effective potential, whereas there is no point of stable equilibrium in an electrostatic field. The important result due to KLL has been also derived by Jung [35] through a Fokker–Planck equation approach [36]. Here, we briefly provide an alternative derivation of this result through the Langevin dynamics approach with special reference to the applications we will be dealing with in this Letter. The variation of the applied field in space is smooth but otherwise arbitrary. Such type of fields are usually applied experimentally to cold atoms, where a very high degree of control is possible. In passing, we also mention that Gommers et al. [37] have investigated the averaged velocity of the atoms in a nonadiabatically driven optical lattice system subjected to a phase modulation of the lattice beams. They found that the resonant activation leads to resonance as a function of driving frequency in the flow of atoms through the periodic potential. They have also shown that the resonance appears as a result of the intricate interplay between deterministic driving and fluctuations. Furthermore, it has been predicted that by changing the frequency of driving, it is possible to control the direction of diffusion. It has been reported in literature that in the presence of nonadiabatic driving, the lifetime of the particle in the potential well can be significantly monitored, an aspect that provides various physically interesting phenomenon. Also, nonadiabatic driving may result in a significant enhancement of the activation rate. Brownian motors have also been realized with nonadiabatically driven Brownian particles. We would like to mention that the rectification of fluctuations for nonadiabatically driven Brownian particles have already been demonstrated in previous works [38,39]. Borromeo and Marchesoni [6] have illustrated that the mobility oscillations owing to the effect of a high-frequency modulation on the output of a periodic device is an oscillating function of the amplitude-to-frequency ratio of the high-frequency drive. In contrast to common observation based on the linear response theory, they showed that the effect of high-frequency modulations can control the response of slow motion.

2. Motion in a high-frequency periodic potential: Model

In this section, the dynamics of a classical particle moving in one dimension under the influence of a rapidly oscillating force (i.e., periodic in time) has been studied using the KLL time window [or multiple scale perturbation theory (MSPT)]. The Brownian particle (of unit mass) that is driven by a rapidly oscillating external field (with frequency Ω) can be modeled by a Langevin equation,

$$\ddot{x} = -V_0'(x) - \gamma\dot{x} + F + \xi(t) + A\cos(\Omega t), \tag{1}$$

where γ is the dissipation constant in the Markovian limit, $V_0(x)$ is the confining potential, F is the static force, A and Ω are the amplitude and the frequency of the external high-frequency impulse, respectively. Here, $\xi(t)$ is the Langevin force, the statistical properties of which can be described by $\langle\xi(t)\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = 2\gamma k_B T\delta(t-t')$ where T is the thermal equilibrium temperature and k_B is the Boltzmann constant. Derivatives with respect to coordinate and time of a function $f(x,t)$ here are described by f' and $\dot{f}$ respectively, while the two overhead dots denote double time derivative. The assumption of one-dimensional motion for the coordinate x does not detract from the general conclusion discussed below. Although, whenever the interaction with an environment affects the dynamics of a system on the time scale on which the state of a system is focused, components of randomness may influence the system dynamics giving rise to a stochastic time evolution [40], and hence, the solution of Eq. (1) with time-dependent potential is very hard to achieve and is usually attained numerically or by means of some approximate methods. A distinct time-scale based separation of the particle's motion into slow and fast variables is made feasible, provided the period of external force is small in comparison to all other time scales associated with the problem. Such a separation is a consequence of the fact that the periodic force changes its sign much more rapidly as compared to the time taken by the particle to re-assume a new set of coordinates. In the other word, in a given period, the periodic force has an insignificantly small contribution to the acceleration. Therefore, the limit of large frequencies (or small periods) becomes pertinent under such circumstances and the actual motion of the system then consists of a rapid motion in proximity to the trajectory of the slow dynamics.

To apply the MSPT solving technique in presence of a generalized (coordinate dependent/independent) rapidly oscillating external force field, we consider a general form of Eq. (1)

$$\ddot{x} = -V_0'(x) - \gamma\dot{x} + F + \xi(t) - V_1'(x, \Omega t), \tag{2}$$

In order to use the MSPT to solve Eq. (2), we introduce the fast time variables $\tau = \Omega t, \epsilon = 1/\Omega$ (where Ω is the frequency of the fast oscillating field and ϵ is the smallness parameter), so that $t = \epsilon\tau$ and τ can be treated as independent variables. We now consider the following expansion of x:

$$x = \sum_{n=0}^{\infty} \epsilon^n x_n(t,\tau). \tag{3}$$

Using the above equation, one can write

$$\frac{d}{d\tau} = \frac{\partial}{\partial\tau} + \epsilon\frac{\partial}{\partial t}, \tag{4}$$

and Eq. (2) without the Langevin force becomes

$$\frac{d^2x}{d\tau^2} + \gamma\epsilon\frac{dx}{d\tau} = -\epsilon^2[V_0'(x) + V_1'(x,\tau)] + F\epsilon^2. \tag{5}$$

By exploiting Eq. (4), Eq. (5) can be written as

$$\left(\frac{\partial^2}{\partial\tau^2} + 2\epsilon\frac{\partial^2}{\partial t\partial\tau} + \epsilon^2\frac{\partial^2}{\partial t^2}\right)\sum_{n=0}^{\infty}\epsilon^n x_n(t,\tau) + \gamma\epsilon\left(\frac{\partial}{\partial\tau} + \epsilon\frac{\partial}{\partial t}\right)\sum_{n=0}^{\infty}\epsilon^n x_n(t,\tau)$$
$$= -\epsilon^2[(V_0'(x_0) + V_1'(x_0,\tau)) + \epsilon x_1(V_0''(x_0) + V_1''(x_0,\tau)) + \ldots] + F\epsilon^2. \tag{6}$$

We are now in a position to equate the terms which are of order ϵ from both sides of Eq. (6). Considering up to sixth order correction, Eq. (2) can be written as [13–15]

$$\ddot{x}_0 = -V_{eff}'(x_0) - \gamma\dot{x}_0 + F + \xi(t), \tag{7}$$

where the effective potential takes the following form

$$\begin{aligned} V_{eff}' = V_0'(x_0) &+ \frac{1}{\Omega^2}\left[\overline{\int^{\tau} d\tau_1 V_1''(x_0,\tau_1)\int^{\tau} d\tau_1 V_1'(x_0,\tau_1)}\right] \\ &- \frac{\gamma}{\Omega^3}\left[\overline{\int^{\tau} d\tau_1 V_1''(x_0,\tau_1)\int^{\tau}\int^{\tau_1} d\tau_1 d\tau_2 V_1'(x_0,\tau_2)}\right] \\ &+ \frac{V_0'''(x_0)}{2\Omega^4}\left[\overline{\left(\int^{\tau}\int^{\tau_1} d\tau_1 d\tau_2 V_1'(x_0,\tau_2)\right)^2}\right] \\ &+ \frac{1}{2\Omega^4}\left[\overline{V_1'''(x_0,\tau)\left(\int^{\tau}\int^{\tau_1} d\tau_1 d\tau_2 V_1'(x_0,\tau_2)\right)^2}\right]. \end{aligned} \tag{8}$$

We must note that the Ω^{-2} and Ω^{-3} terms vanish if the external periodic force is coordinate independent (i.e., only time dependent external force field.) In this case, we have considered only a time dependent periodic force. So it is very essential for us to consider the Ω^{-4} or higher order terms to get the effective potential, as the