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Entropy generation: From outside to inside!

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ABSTRACT

A recent analysis of the Carnot's results has pointed out that natural systems may not convert all the inflow of energy to do work. Some energy will be used to maintain the systems' internal processes. These exergy flows appear as the heat exchanged with a second thermostat of a thermodynamic engine. In this Letter a calculus of this internal irreversibility is developed using the entropy generation approach. The obtained results are exemplified in the analysis of superconductivity.

1. Introduction

The constructal theory by Adrian Bejan describes how natural as well as engineered systems develop and evolve to result in scale-free patterns, most notably power laws [1]. The constructal law describes both internal interactions within a system as well as interaction of a system with its environment. The physical basis of the constructal law can be found from the entropy generation approach. The constructal law portrays the system from inside while the entropy generation views the system from outside, yet both viewpoints are equivalent since it is the same quanta that traverse from the environment to the system or vice versa when the system evolves to attain balance in its environment. Thus when the maximum entropy generation from the environment is evaluated it corresponds to the minimum entropy generation from the system, with the consequent optimization of the flows, as required by constructal law. Along the optimal path toward the thermodynamic balance free energy is consumed in the least time which yields the ubiquitous natural patterns [2].

In 1824, Nicolas Léonard Sadi Carnot [3] introduced an ideal engine which operates on a cycle in a reversible way without dissipation. Such system could convert the absorbed heat in work, without any energy loss, because, apparently, it has no irreversibility. But analyzing this ideal system Carnot proved that [3]:

1. All ideal engines operating between the same two thermal baths (thermostats) of temperature T_1 and T_2, with $T_1 > T_2$, has the same ideal efficiency $\eta_C = 1 - T_1/T_2$.
2. Any other engine has an efficiency η such that it is always $\eta < \eta_C$.

Consequently, the efficiency of a reversible Carnot's cycle represents an upper limit of the thermal efficiency for any heat engine working between the same temperature limits [4]. Real machines operate on thermodynamic processes and take place in finite-size devices in finite-time: any change of state will consume free energy; consequently, it is irreversible. A great number of studies has been developed on them and they has always proven the Carnot's results [5–14]. Carnot's conclusion is a general result for any natural or manmade system.

Starting from the Gouy–Stodola theorem [15–18], mathematical thermodynamic analysis of the Carnot's result has been developed [19] by using the entropy generation approach [20–23]. The result obtained is that the systems may not convert all the absorbed energy because they must use a part of it to maintain the process they are doing. The energy which can be really used by a system in called exergy. From any reference frame, external to the systems, these exergy flows appear as the heat exchanged with a second thermostat of a thermodynamic engine [21]. This heat seems to be lost and not used by the system, while it is really used by the process [19]. This effect has been defined 'internal irreversibility' [21].

This internal irreversibility is related to the need to maintain the process [20–23] and it is due to the existence of the internal forces and flows which require energy to occur. Without these internal flows the systems cannot sustain their process. Consequently, a part of the energy absorbed by the systems must be converted in internal flows and cannot be used to convert the absorbed heat in 'useful' work [19]. In other words, when free energy is consumed, the quanta absorbed by the system from its surroundings are not only emitted free as heat but some are also bound to the developing system's internal interactions. Only when the system has matured to a steady-state thermodynamic balance within its surroundings, absorption and emission match each other. Accordingly, exergy flows can be understood to power developmental or evolutionary processes, as explained by Annila et al. [2,19,24–32]. Constructal theory expresses these results, allowing us to design systems [33–36]: nature is the first engineer in the history!

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But, the flows are related both to the interactions of the systems with the environment (external irreversibility) and to internal irreversibility. So, it is interesting to study how it is possible to evaluate internal irreversibility and how it is related to the entropy generation extrema approach [20].

In this Letter this subject will be developed, analyzing the relations between the interaction among open or closed systems, their internal irreversibility and their interaction with the environment. The superconductivity will be analyzed as an example of application of the results obtained.

2. System and environment: interactions in terms of forces

Let us consider a real open or closed system, in nature or man-made [4,36,37]. For such system, it is possible to write the kinetic energy theorem [38]:

$$W_{es} + W_{fe} + W_i = \Delta E_k \tag{1}$$

where W_{es} is the work done by the environment (external to the system) to the system, the work done by external forces to the border of the system, W_{fe} is the work lost due to external irreversibility, E_k is the kinetic energy of the system, W_i is the internal work, such that [38]:

$$W_i = W_i^{rev} - W_{fi} \tag{2}$$

with W_i^{rev} reversible internal work and W_{fi} work lost due to internal irreversibility. Moreover, the following relation must be taken in account [38]:

$$W_{se} = -W_{es} - W_{fe} \tag{3}$$

where W_{se} is the work done by the system to the environment, the work done by internal forces to the border of the system. Consequently, it is possible to obtain the three equivalent following formulation of the first principle as usually used in the designing of the energy systems and the analysis of the engineering thermodynamic systems:

$$Q - W_{se} = \Delta U + \Delta E_k$$
$$Q - W_i = \Delta U$$
$$Q - W_t = \Delta H$$

with W_t the technical work, U the internal energy of the system and $H$the enthalpy [4,36,37].Now, considering the Gouy–Stodola theorem the work lost due to external irreversibility can be written as [4,15–18]:

$$W_{fe} = T_0 S_g \tag{5}$$

where T_0 is the environmental temperature and S_g is the entropy generation defined as [4,8–10,15–23]:

$$\begin{aligned} \Delta S &= \Delta S_{rev} + S_g \\ \Delta S_{rev} &= \int_{is}^{fs} \frac{\delta Q}{T}\Big|_{rev} \end{aligned} \tag{6}$$

where ΔS_{rev} is the entropy variation of the system from the initial (is) and the final (fs) state on a reversible path, with δ the elementary small change of a path function, Q heat exchanged during the process on the reversible path and T temperature [20].

Now, considering the previous relations, it is possible to obtain the link between the internal and the external works lost due to irreversibility. Indeed, it follows:

$$\begin{aligned} W_{fi} - W_{fe} &= W_i^{rev} + W_{es} - \Delta E_k \Rightarrow W_{fi} - T_0 S_g \\ &= W_i^{rev} + W_{es} - \Delta E_k \end{aligned} \tag{7}$$

Now, considering that the total work lost due to irreversibility was obtained as follows [39]:

$$\begin{cases} W_\lambda = \int_V \left(\int_0^\tau dt\, T v\sigma - \varphi \right) dV = (T - T_0) S_g \\ W_\lambda = W_{fi} - T_0 S_g \end{cases} \tag{8}$$

where $\sigma = \delta^2 S_g / dt dV$ is the entropy production density, φ is the dissipation function, v is the specific volume, τ is lifetime of the process and V is the control volume of the system [20], it follows:

$$W_{fi} = \int_V \int_0^\tau dt\, T v \sigma dV = T S_g \tag{9}$$

This relation represents the work lost due to internal dissipation: these internally dissipated quanta are bound in internal interactions that form when the system develops toward thermodynamic balance with its surroundings.

Consequently, the work lost due to internal dissipation can be evaluated using the entropy generation, but relating it to the temperature T of the system.

3. An example: the high-T_c superconductivity

The discovery of high-T_c superconductivity has given new interest in the application of the type-II superconductors [39]. They are characterized by two critical magnetic fields and a particular behavior:

1. For applied fields less than the lower critical field, they have a Meissner effect, and the entire sample behaves as a type-I superconductor;
2. For applied fields greater than the upper critical field, there occurs a complete penetration of the magnetic field and the material is in a normal conducting state;
3. For field intensities between the two limit magnetic fields, there is a partial penetration of the magnetic field, and the field lines are confined to flux tubes, named vortices [40–45]. Along this vortex lattice the material has a normal resistivity, while its surrounding behaves as a superconductor. These vortices are interesting because they allow to test many statistical phenomena [40–45].

In order to use the results here obtained to the high-T_c superconductivity, the approach used in Ref. [46] will be considered. The analysis has been carried out by using a dimensionless quantity [46]:

$$\vartheta = \frac{n}{8\pi\alpha\lambda^3} \left(\frac{h}{2e} \right)^2 \tag{10}$$

where n is the number of vortexes for unit length of superconductor, λ is the London penetration length, $\hbar/2e$ is the unit flux of quanta and α is defined such that $\alpha\lambda^2$ is a confining harmonic energy. This quantity represents the ratio between the thermal and the harmonic energy.

Now, considering an external force in the x direction, $\boldsymbol{F} = -\alpha x\, \boldsymbol{x}/x$, the entropy generation can be evaluated by using its definition and the stationary solutions of the Fokker–Planck equation, obtaining:

$$S_g = \frac{\tau}{T_0} \int \sum_i f_i \left(f_i + D_i \frac{\partial}{\partial x_i} \right) P_\gamma(\mathbf{x}, t) d\mathbf{x} = \frac{\tau}{T_0} \int \sum_i f_i J_i(\mathbf{x}, t) d\mathbf{x} \tag{11}$$

with

$$\begin{aligned} P(x,t) &= \frac{1}{4\vartheta\lambda^3} (x_e^2 - x^2) \\ x_e &= \lim_{t\to\infty} \bar{x}(t) = \sqrt[3]{\frac{3D\lambda}{\alpha}} \end{aligned} \tag{12}$$

with