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Inverse error-diffusion using iterated conditional modes

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ABSTRACT

Error-diffusion (ED) is one conventional halftoning technique that converts a gray-level image into a halftone. For further processing the ED halftones, it is often necessary to estimate the original image from the halftone: the inverse of ED. We propose to calculate iterated conditional modes (ICM) for the maximum a posteriori (MAP) solution of inverse ED. The ICM always searches for a better estimate in the valid image space. It requires only local computation and is applicable to any type of the MRF model used for the original gray-level images. Experimental results for common standard images are given to show that our ICM performs well and is more flexible than the descent-projection (DP) approach.

1. Introduction

In modern communication system, the multimedia signal, especially the image signal, plays an important role and becomes more and more popular. In the system, the image can be present in the form of *halftones*, for example, under the circumstance where the printing or displaying of images is required and the number of inks or colors in the printing/displaying devices is small. A halftone is the output of the process called *halftoning* that converts a given *continuous-tone* image into an image with very limited number of levels, for example, binary [2]. A halftone is hence a state-resolution reduced version of the original continuous-tone image. It is just one visual approximate of the original image and is too rough to be used for further processing in many applications. These applications can include re-halftoning with different dot patterns or dot sizes, resizing the image, enhancing the image contents, and etc. For the further processing, it is often necessary to perform some kinds of *inverse halftoning* that inverts the effect of halftoning, that is, gets an estimate of the original image from a given halftone. Inverse halftoning has much practical value and has attracted considerable attention of many researchers (see, for instance, the recent papers [6,8,14–17,30,31]). The details of methods for inverse halftoning could be quite different from one another, depending on the various techniques used for the halftoning. Conventionally, two main halftoning techniques are the *dithering* and the *error-diffusion* (ED) [20,39]. ED halftones are very common since they provide much higher image quality than dithering halftones. This paper focuses on the inverse of ED.

1.1. Background

To reduce the state-resolution of a continuous-tone image, the ED involves a quantizer that assigns to its input value one of several, say binary, halftone values. Since the quantization is a nonlinear operation, the solution to the inverse ED is not unique. A number of methods have been proposed to overcome the non-uniqueness of inverse ED. They can be briefly classified into two types: the approach based on the ED model (model-based inverse, MI) and the approach *not* based on the ED model (non-model-based inverse, NMI). In MI, the inverse took advantage of the information on the ED model employed to produce the halftone. The model parameters (i.e., the kernel of ED model) were thus used in solving the inverse problem [14,15,18,21,31,35,40,41]. On the other hand, the NMI regarded the inversion as some kind of denoising or filtering. Simply based on the halftone image, the filter was designed to smooth the halftone but preserve important edges as much as possible [4–6,8,16,17,22–25,27,28,30,42]. Such denoising ignored or was blind to the ED model. It worked without the knowledge of the model parameters. However, to obtain the best performance, we should use as much as possible the information we can use, including the model information. The ignorance of model information inevitably limits the best performance the NMI can achieve. Often, the methods of NMI highlighted the fast computation speed or low complexity, and sacrificed some of their performance. In fact, some methods of NMI utilized indirectly the model information, since they used a set of training halftones to

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design the denoising filter and the halftones were generated from a ED process with known model parameters [4–6,16,17,22–24,27,28]. This fact reflects exactly the importance of the model information. This paper focuses on the MI approach.

In the spectrum of the MI, there are two main strategies for inverse ED. One formulated the inverse ED as the rule of projection onto convex sets (POCS) [14,18,40]. The *a priori* information on the ED model and original image were expressed in the form of several convex sets. Then, intermediate estimates of the original image were obtained by iteratively projecting onto these convex sets. The theory of convex sets guarantees the final convergence of the estimate. On the other hand, the other strategy described the inverse ED as a stochastic framework [15,35,41], and took the advantage of some kinds of statistical optimization principles. Fung and Chan [15] used a simulated annealing procedure to inverse the ED of a color image. The procedure iteratively minimized the cost function defined as the difference between the given halftone and the halftone obtained from error-diffusing an intermediate estimate of the color image. The ED model parameters were required in the diffusing and a stochastic mechanism based on the annealing *temperature* permitted the estimate to escape from a local minimum of the cost function. In the procedure, there was no exploration of the spatial properties of the continuous-tone image. Wong [41] estimated the continuous-tone image in an iterative manner and achieved very good results. In each of the iteration, the estimation performed a filtering followed by a projection. The filtering consisted further of two steps: a linear low-pass filtering preceding a nonlinear statistical smoothing, which was designed according to the local variation of the continuous-tone image. In addition, the projection was a maximum a posteriori (MAP) estimation that selected the most likely continuous tone given an observed halftone pixel. The selection needs the ED model information. We note that the statistical smoothing and the MAP projection simply regarded the continuous-tone image as an array of independently identically distributed (iid) random pixels. This assumption of independence is just an ideal condition. The pixels in an image are generally dependent. Stevenson [35] took this dependence into account and modeled the continuous-tone image as a Huber–Markov random field (HMRF). Then, the inverse ED was treated as an MAP problem that searched for the most likely continuous-tone image given the observed halftone. The ED model information was a necessary part of the MAP formulation. This Bayesian modeling was indeed very interesting. Stevenson [35] resolved the MAP problem via iterative two phases: the descent phase and the projection phase. The former seeks an estimate with the minimum energy that is determined by the optimal step in the descent direction. However, the estimate found is not guaranteed to fall in the valid image space. A second phase of projection is thus necessary. By applying a rule similar to the MAP projection in [41], the projection phase constrains the estimate into the valid image space. In such descent-projection (DP) approach, the minimization in the descent phase is based on a second-order Taylor series expansion, which involves the Hessian matrix of the objective function to be minimized. The elements of the Hessian matrix entail further the second-order derivative of the Huber function. However, the second-order derivative of the Huber function does not exist at the Huber threshold T (please refer to Section 3 for the definition of T). Moreover, the results of [35] were not evaluated by some objective indices, for example peak signal-to-noise ratio (PSNR), which could be used for fair comparison with other approaches.

1.2. The proposed method

To deal with the inter-pixel dependence of an image, we also adopt an MRF to model the continuous-tone image and pursue an MAP solution. However, differently from the DP approach [35], we suggest to calculate iterated conditional modes (ICM) for solving the MAP problem. An abridged early-stage version of this work was given in Ref. [19]. The method of iterated conditional modes (ICM) was introduced by Besag [3]. It is a clever solution to the MAP problem. It avoids the computational burden inherent in the MAP modeling of image, and hence become popular in the community of stochastic signal analysis, for example, see some recent papers [1,7,9,12,13,26,32–34,36–38,43]. Our ICM technique adapts to the ED model. It always searches for a better estimate (aimed at the MAP solution) in the *valid* image space and hence prevents the difficulty of going beyond the valid space, which appeared in the DP approach. Experimental results show that the ICM not only has performance comparable to the DP approach for transient inverse, but also outperforms the DP approach when the restored image needs to be stored (in the integer format of general digital images) for later usage. It is thus more flexible than the DP approach. We also compare the performance of the ICM technique, which belongs to the stochastic MI approach, and some previous methods of the POCS-based MI approach. Such comparison between the stochastic and the POCS-based MI methods would be valuable, but we have not ever seen such comparison in the literature.

This paper is organized as follows: in Section 2 we introduce the error-diffusion (ED), the halftoning operation we are going to inverse. Section 3 is devoted to statistical concepts on which the proposed method is based. In Section 4 we explain the details of the proposed method of iterated conditional modes (ICM). Section 5 gives some typical experimental results, which show the good performance of our method. Finally, in the last section, we present our conclusions.

2. Error-diffusion

Digital halftoning can be viewed as 1-bit quantization. Let $x_s \in [0, G]$ be the gray level of the original image at a given site $s \in L$, where L is the lattice system on which the image is defined and G is the maximum possible gray level, for example, $G = 255$ for 8-bit gray-scale images. Given x_s, the output of the quantizer, i.e., the corresponding halftone image at the same site, will take a binary value $b_s \in \{0, G\}$. Since the halftone value has only two states, it is certain for general images that the quantized level b_s is different from the input of the quantizer, yielding a quantization error. The halftone $b = \{b_s; s \in L\}$ can approximate the original image $x = \{x_s; s \in L\}$ when being viewed in a proper distance.

As one of the most common types of digital halftoning [39], the error-diffusion (ED) is based on the simple principle that the quantization error at each site should be diffused to its neighboring pixels to affect their quantization. Specifically, for a given site $s \in L$, the halftone value is the output of the quantization $Q(\cdot)$ defined by

$$b_s = Q(x'_s) = \begin{cases} G, & \text{if} \quad x'_s \geqslant G/2 \\ 0, & \text{else} \end{cases} \tag{1}$$

with

$$x'_s = x_s + \sum_{t \in R_s} a_t e_t \tag{2}$$

and

$$e_s = x'_s - b_s, \tag{3}$$

where e_s is the error signal at site s, a_ts are the filter coefficients and R_s is the region of support for the coefficients. Conventionally, the region R_s is causal, that is, the error is diffused to the right and below. The quantization of (1) is performed from site to site in the order of raster scan. The effect of ED can be interpreted as a way of keeping the local average intensity of the halftone image as close