



# How does the presence of another isotopic species affect the ratio of the forward to backward emission fluxes in a slab geometry



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## ABSTRACT

Emission from a mixture of two isotope species in slab geometry is examined for slab thickness that is a multiple of half-wavelength of the resonant radiation. The forward and backward outgoing fluxes for different detuning between the two isotope species resonance frequencies are computed. The values of the initial population giving equal forward and backward fluxes or making the ratio of these two quantities an extremum are obtained. The characteristic temporal features of the field intensities and of their spectral distributions at the previous critical values are calculated.

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## 1. Introduction

Emission from a mixture of two isotopes has been a topic of continuing interest since the early work on laser isotopes separation [1–5]. In a recent publication [6], for 1D geometry, the spatio-temporal electrodynamics of a mixture of two isotope species, where initially only one of the species was excited (partially or fully) while all atoms of the other species are left in the ground state was examined. In particular, the general features of the temporal development of the emission from the system at the exit planes, and the spatial distribution of the degree of excitation for each of the two atomic species at different times were investigated as function of the detuning between the two isotopes resonance frequencies and the degree of the initial excitation of the excited species.

In a separate study of radiation from an ensemble of a single species [7], it was shown that there was extreme ultra-sensitivity to initial conditions of the ratio of the forward to backward emission fluxes in a slab having a thickness which is a multiple of half-wavelength when the atoms of the ensemble are nearly all inverted: this ratio can vary by nearly one and a half order of magnitude for a slight variation in the initial atomic excitation.

In this paper, the characteristics emission features of the two species mixture are obtained for the case that the slab thickness is a multiple of half the wavelength of the radiation corresponding to the free atomic resonance frequency, the system is initially coherently excited, and the inversion is almost complete.

The main results obtained in this manuscript can be summarized as follows:

- The two-species system is also ultra-sensitive to the initial value of the population difference of the excited species [8–14].
- The values of the initial population difference where the forward and backward emission fluxes are equal and where either flux is extremal are shifted from their values for the system with only one species present and these differences are computed as functions of the detuning between the resonance frequencies of the two species.
- The contrast function, defined as the ratio of the maximal to minimal values of the ratios of the forward to backward fluxes, is shown to be dependent, as well, on the detuning between the resonance frequencies of the two isotopic species resonance frequencies (see graphical abstract). The value of the contrast function is found to be largest for the case of a single species present.

The theoretical model and the numerical techniques used to obtain the present results follow structurally the same algorithm as that used in deriving the results in [6]. In the present calculation, the system is taken to be initially coherently prepared. The accuracy of the algorithm used has been, as well, enhanced relative to that used in [6] (by increasing the size of the functions basis) to properly deal with the minute variations in the initial conditions required for the different cases considered here. All computations performed use the basis formed by the eigenfunctions of the 1D Lienard–Wiechert kernel [15–21].

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The paper is organized as follows: in Section 2, for the purpose of ensuring that this presentation be self-contained, I review the expressions of the Maxwell–Bloch equations for the two isotopes mixture in the normalized coordinates suitable for the present problem; in Section 3, I summarize the mathematical method used for reducing the partial differential equations representing the MB set into an infinite set of coupled first order ordinary differential equations for the expansion coefficients in the basis formed by the eigenfunctions of the 1D Lienard–Wiechert kernel, and I give explicitly the initial conditions for these expansion coefficients for the case of initial coherent excitation; in Section 4, I give the results of the numerical computations. In Section 5, I summarize and conclude.

## 2. Maxwell–Bloch equations in normalized coordinates

In this Section, I review quickly the form of the Maxwell–Bloch equations for this system when written in normalized variables form, convenient for the present problem. I assume that the mixture is formed by equal concentration of the two isotopes, and that the transition dipole moment for both isotopes are equal.

Defining the normalized variables for a slab of thickness  $2z_0$ , as:

$$Z = z/z_0, \quad T = Ct, \quad \Gamma_1 = \gamma_1/C, \quad \Gamma_2 = \gamma_2/C, \quad u_0 = k_0 z_0,$$

$$\Omega_{c,1,2,L} = \omega_{c,1,2,L}/C,$$

where  $\Omega_{c,1,2,L}$  are respectively the normalized electric field carrier frequency, the atomic transition frequency for species 1, the atomic transition frequency for species 2, and the Lorentz shift. In this system of units, all quantities are normalized to the parameter of interatomic cooperativity  $C = \frac{4\pi N \mu^2}{hV}$ , where  $N$  is the total number of atoms,  $V$  is the slab volume, and  $\mu$  is the reduced dipole moment of the atomic transition (its normalization is uniquely determined when given as function of the isolated atom decay rate, see below). The relaxation decay rates  $\gamma_1$ ,  $\gamma_2$  refer respectively to the longitudinal decay rate, and the resonant transverse decay rate. The normalized transverse resonant decay rate  $\Gamma_2$  is due to the instantaneous dipole–dipole interaction between identical atoms, and the value of the normalized Lorentz shift is equal to  $1/3$ . The isolated atom decay rate  $\gamma_1 = \frac{4}{3} \varphi^2 k_0^3 / h$  specifies the longitudinal decay rate of each of the species. To ensure that the independent particles model is valid, I shall consider only the cases where the detuning (denoted  $\Delta$ ) between the resonance frequencies is at least  $4\Gamma_2$ .

The Maxwell–Bloch equations in 1D are given in these units, for  $\xi = Cz_0/c \ll 1$ , where  $c$  is the speed of light in vacuum, by

$$\frac{\partial \chi_1(Z, T)}{\partial T} = -[i(\Omega_1 - \Omega_c) + \Gamma_T - i\Omega_L n_1(Z, T)] \chi_1(Z, T) + \frac{i}{2} n_1(Z, T) \psi(Z, T), \quad (1a)$$

$$\frac{\partial \chi_2(Z, T)}{\partial T} = -[i(\Omega_2 - \Omega_c) + \Gamma_T - i\Omega_L n_2(Z, T)] \chi_2(Z, T) + \frac{i}{2} n_2(Z, T) \psi(Z, T), \quad (1b)$$

$$\frac{\partial n_1(Z, T)}{\partial T} = -i[\chi_1^*(Z, T) \psi(Z, T) - \chi_1(Z, T) \psi^*(Z, T)] + \Gamma_1(1 - n_1(Z, T)), \quad (2a)$$

$$\frac{\partial n_2(Z, T)}{\partial T} = -i[\chi_2^*(Z, T) \psi(Z, T) - \chi_2(Z, T) \psi^*(Z, T)] + \Gamma_1(1 - n_2(Z, T)), \quad (2b)$$

$$\psi(Z, T) = iu_0 \int_{-1}^1 dZ' [\chi_1(Z', T) + \chi_2(Z', T)] \exp(iu_0|Z - Z'|), \quad (3)$$

where  $\Gamma_T = \Gamma_2 + \frac{\Gamma_1}{2}$ , and  $\chi_{(1,2)}$  (complex) and  $n_{(1,2)}$  (real) describe respectively the atomic polarization density and the degree of exci-

tation of each species (in the present paper,  $n_i = 1$  if all atoms of species  $i$  are in the ground state and  $n_i = -1$  if all atoms of species  $i$  are excited), and  $\psi$  represents the normalized Rabi frequency of the complex total electric field envelope.

## 3. Eigenfunction decomposition

The system described by Eqs. (1)–(3) is solved by expanding each of the quantities  $\psi(Z, T)$ ,  $n_i(Z, T)$ ,  $\chi_i(Z, T)$  in the basis formed by the eigenfunctions of the integral equation

$$\Lambda_s \varphi_s(Z) = \frac{u_0}{2} \int_{-1}^1 dZ' \exp(iu_0|Z - Z'|) \varphi_s(Z'). \quad (4)$$

The expressions of the normalized eigenfunctions are respectively:

$$\tilde{\varphi}_m^o(Z) = \frac{1}{\sqrt{N_m^o}} \sin(v_m^o Z), \quad (5)$$

$$\tilde{\varphi}_m^e(Z) = \frac{1}{\sqrt{N_m^e}} \cos(v_m^e Z), \quad (6)$$

where the normalization constants are

$$N_s^o = 1 - \frac{\cos^2(v_s^o)}{iu_0}, \quad (7)$$

$$N_s^e = 1 - \frac{\sin^2(v_s^e)}{iu_0}, \quad (8)$$

and the wavevectors ( $v_s^o$ ,  $v_s^e$ ) are solutions of the transcendental equations

$$\cot(v_s^o) = i \frac{u_0}{v_s^o}, \quad (9)$$

$$\tan(v_s^e) = -i \frac{u_0}{v_s^e}. \quad (10)$$

The integral equation eigenvalues are given by:

$$\Lambda_s^{o,e} = \frac{iu_0^2}{u_0^2 - (v_s^{o,e})^2}. \quad (11)$$

The integral equation eigenfunctions obey pseudo-orthonormal relations and form a complete set of bases functions for all functions over the interval:  $-1 \leq Z \leq 1$ .

In this algorithm, the various dynamical variables are decomposed in the eigenfunctions basis, as follows

$$\psi(Z, T) = \sum_s e_s^o(T) \tilde{\varphi}_s^o(Z) + \sum_s e_s^e(T) \tilde{\varphi}_s^e(Z), \quad (12)$$

$$n_i(Z, T) = \sum_s \eta_{i,s}^o(T) \tilde{\varphi}_s^o(Z) + \sum_s \eta_{i,s}^e(T) \tilde{\varphi}_s^e(Z), \quad (13)$$

$$\chi_i(Z, T) = \sum_s p_{i,s}^o(T) \tilde{\varphi}_s^o(Z) + \sum_s p_{i,s}^e(T) \tilde{\varphi}_s^e(Z), \quad (14)$$

where the tilde over the eigenfunction is used to indicate that I am using the normalized eigenfunctions in the expansions.

Combining Eqs. (3) & (4), one deduces:

$$e_s^{o,e}(T) = i2\Lambda_s^{o,e} (p_{1,s}^{o,e}(T) + p_{2,s}^{o,e}(T)) \quad (15)$$

The set of partial differential equations given by Eqs. (1)–(3), augmented by Eq. (8) then combine to give an infinite dimensional set of coupled nonlinear ordinary differential equations for the expansion coefficients  $p_{i,s}^{o,e}(T)$  &  $\eta_{i,s}^{o,e}(T)$ .

The initial conditions for a system where the species initially inverted was coherently excited and the other species is left in the ground state are:

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