Journal of Electron Spectroscopy and Related Phenomena 205 (2015) 66–73

Contents lists available at ScienceDirect

Journal of Electron Spectroscopy and Related Phenomena

journal homepage: www.elsevier.com/locate/elspec

Entanglement manifestation in spin resolved electron–electron scattering

O.M. Artamonov[a,*], S.N. Samarin[a,b], A.N. Vetlugin[a], I.V. Sokolov[a], J.F. Williams[b]

[a] Institute of Physics, St Petersburg State University, St Petersburg, Russia
[b] School of Physics, the University of Western Australia, Crawley, Perth 6009, WA, Australia

ARTICLE INFO

Article history:
Received 23 March 2015
Received in revised form 29 June 2015
Accepted 10 August 2015
Available online 19 August 2015

PACS:
79.20.Uv
03.67.Mn
34.80.Pa

Keywords:
Electron scattering
Spin polarization
Entanglement state

ABSTRACT

The polarization vector **P** of scattered electrons interacting with a polarized target electrons is compared with the entanglement (or non-separability) of the electron states of the interacting electron pair. The separability S is defined as a linear function of the von Neumann entropy. The shapes of the functions $P(\theta,\Omega,\varphi)$ and $S(\theta,\Omega,\varphi)$ are similar and simultaneously achieve their maximum value at the scattering angle θ values close to 0 and π and simultaneously tend to zero in the case of symmetric scattering at $\theta \approx \pi/2$. In the latter case the scattered electrons are described by an asymmetric spin part of the wave function, which by definition corresponds to the spin entangled ($S \approx 0$) electron states of the interacting electron pair. Comparison of the model calculation results with experimental results of the spin polarized electron spectroscopy of the ferromagnetic solid shows qualitative agreement. The analytical expression relating polarization and separability of the two interacting particles enables use the measured polarization of scattered electrons for estimation of the spin-entanglement or separability of the two particle systems.

1. Introduction

Entangled quantum states [1], the wave function of which cannot be expressed as a product of the wave functions of all participating particles, have strong internal correlation. Because of this correlation any measurements performed on one of the particles lead to changes in the wave function of the other particle even if this second particle is very far from the first one [1,2]. For the two-particle systems in the entangled state the name “EPR-pair” is often used in memory of the famous paradox formulated by Einstein, Podolsky and Rosen, which is still discussed [3]. Many investigations, mainly concerning photon states, are devoted to the problems of detecting of the entangled states and manipulation of the entangled *states*; see review papers [4,5]. In the case of electron states certain progress has been achieved for mesoscopic solid samples, where the sample dimensions are commesurate with the ballistic mean free path of electrons in solids [6–8]. We are not aware of successful investigations of the electron EPR pairs of free electrons. We intend to consider the role of spin entangled electron states in spin polarized electron spectroscopy of ferromagnetic solids.

The entanglement appears to be one of the key properties of the quantum mechanics: quantum computing, quantum cryptography and quantum teleportation occurred to be possible because of it, but it is quite difficult to detect the presence of entangled states and manipulate them. For the two-particle quantum systems the spin entanglement of the states means that spin of one of the particles is rigidly correlated with spin of the other particle. This correlation could be observed if spin projection of one of the particles belonging to the spin-correlated pair was measured by some “spin-detector” while the spin projection of the other particle was measured by another detector. The space between the detectors and the time of measurements may be arbitrary if one neglects interaction with the surroundings. But such experiment looks unlikely because of the very low efficiency of the known spin-detectors. We consider two-particle electron systems entangled in spin noting that for spin-entangled pairs the meaning of description of spin-characteristics of individual particles of the pair is lost. The electron spin states $\left|\chi\right\rangle$ are given by a coherent superposition of two states with the spin projection on chosen directions “up” $|+\rangle$ and “down” $|-\rangle$. All that we can measure for an individual electron is to determine the direction of the spin projection on the chosen axis. In the measurement process decoherence of the state occurs and it

---

[*] Corresponding author. Tel.: +7 8124329157.
E-mail addresses: artaoleg@gmail.com (O.M. Artamonov), sergey.samarin@uwa.edu.au (S.N. Samarin).

http://dx.doi.org/10.1016/j.elspec.2015.08.007
0368-2048/

is impossible to measure the spin projection of the same electron on another axis. It is only possible to determine the mean value of the spin vector for an ensemble of electrons being in the same state. An ensemble of electrons is described by the polarization vector. The degree of polarization P, i.e. the expectation value of the Pauli spin operator, or the length of **P**-vector is unity in the pure spin state while a partially polarized beam represents a statistical mixture of different spin states.

Electron systems of two electrons provide a unique possibility to detect spin-entangled states. The two particle system corresponds to the totally entangled singlet state if the spatial part of the wave function is symmetric relative to the coordinate exchange and the spin part of the wave function is anti-symmetric [9]. This is singlet state and the polarization vector of the pair equals zero. When any polarization projection of the scattered electron is being measured then collapse of the two particle wave function occurs and the polarization vector of the detected electron of the entangled pair takes the value up or down in the measurement basis. Detecting of the second scattered electron of the pair by another detector takes the opposite value (down or up in the measurement basis).

The pair of electrons before scattering forms a non-entangled or a separable state. A spin-entangled electron pair may be obtained, for example, as a result of electron–electron scattering, the entanglement degree being dependent on the scattering symmetry and the scattering angle θ. Fig. 1a represents the scattering symmetry which forms the totally entangled electron pair (left panel). Fig. 1b represents the scattering conditions which form the partially entangled (or partially separable) electron pair. After the scattering of the two electrons e_1 and e_2 with anti-parallel polarization (red arrows), the pair of two scattered electrons appears and measurements are made of the polarization of one of the pair (the right one). If one chooses the "symmetric" scattering conditions with the two scattered electrons having equal energy and the scattering angle of $\theta_{\text{lab}} = 45^\circ$ (in the laboratory frame), only electrons of totally entangled pairs will be detected. Since the spin state of the individual electron in the entangled pairs is not defined, the expectation value of the polarization vector of these electrons would be zero. The "degree" of entanglement of the pair of scattered electrons may be changed by changing the kinematics of scattering. If, for example, one chooses then to detect electrons at a very small scattering angle (right panel), then the "separability" would be close to unity and the electron–electron scattering may result in formation of both entangled and non-entangled pairs. If the entangled pairs are not created specially (for example, by choosing the scattering kinematics) then the polarization vector of the electrons measured by one of the detectors has a non-zero value. It follows that the fact of measurement of zero polarization in the scattering of initially polarized electrons forming singlet pairs can be considered as evidence of the presence of entangled states. Thus, for symmetric scattering a zero polarization vector of scattered electrons may be considered as a signature of entanglement.

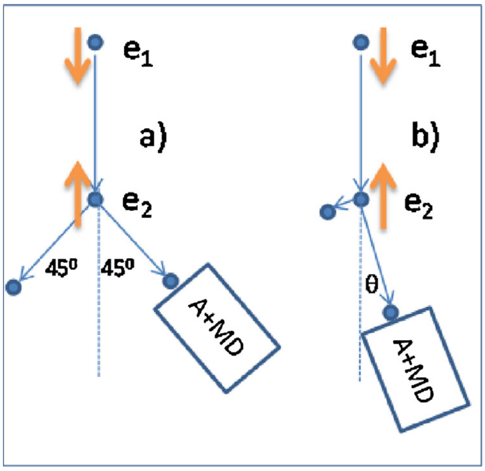

Fig. 1. Two different kinematics allow selection of electrons from totally or partly entangled pairs. The left panel represents the "symmetry scattering" conditions. The right panel represents the common scattering conditions. Also e_1 and e_2 are the primary and the target electrons with anti-parallel polarization (red arrows). The abbreviation "A+MD" means an energy analyzer plus a Mott detector. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Generally, the two particle wave function can be presented as a superposition of separate parts, which consists either of a symmetric spin wave function and an anti-symmetric space wave function or of an anti-symmetric spin wave function and a symmetric space wave function. But, in the case of the electron–electron scattering at the scattering angle $\theta_{\text{lab}} = 45^\circ$ in the coordinate system where one of the electrons is at rest, the conditions are realized in which the two particle wave function has a symmetric space part and an anti-symmetric spin part [10]. For two-particle states the anti-symmetric spin wave function corresponds to a singlet state or to one of Bell maximum entangled spin states [4]. The degree of the interaction in electron–electron scattering is described by the transferred momentum or by the scattering angle. For the pair of electrons detected at given spatial locations r_1 and r_2 a change of scattering angle to obtain "non-symmetric" scattering results in the spin wave function becoming a superposition of singlet and triplet states and the interacting pair is no more in the maximum entangled state.

The main tasks of the present work are: (a) to consider the evaluation of spin wave function of the interacting electron pair with changing kinematics by studying the dependence of the scattered electron polarization on the scattering angle; and (b) to consider the application of the discussed below idealized case to the spin polarized electron spectroscopy of solid surfaces with spin polarized primary electrons and spin polarized valence electrons of a ferromagnetic solid.

2. Conceptual model

Let us consider an idealized case of scattering of free primary electrons with polarization vector "up" from free target electrons with an arbitrarily given polarization (z axis directed upward). The polarization vector of the target electron is directed before scattering at the polar angle Ω relative to the z axis and at the azimuthal angle φ relative to the x axis in the xy plane. We assume that primary and valence electrons are totally polarized, i.e. the polarization vector length is equal to one and in the considered non-relativistic energy range the spin part of the wave function does not depend on the space coordinates [11]. Without loss of generality the spin part of the electron pair wave function $\left|\chi_{12}\right\rangle$ may be presented as a superposition in terms of four Bell two-particle states [4], one anti-symmetric singlet state and three symmetric triplet states. The full wave function $\left|\psi\right\rangle$ then consists either of a singlet spin component and symmetric space wave function or of three triplet symmetric spin components and an anti-symmetric space wave function. To determine the polarization of a state, for example, of the first electron of the pair, it is necessary to construct the density matrix for the system of two interacting particles $\rho = \left|\psi\right\rangle\left\langle\psi\right|$ and take the trace to obtain the one-particle reduced density matrix for the first particle $\rho_1 = \mathrm{Tr}_2(\rho)$ [12]. The spin state of the particle is fully described by the polarization vector **P** (i.e. the expectation value of the Pauli spin operator), which is calculated as an expectation of the projectors on the corresponding coordinate axes $\boldsymbol{P} = \langle\boldsymbol{\sigma}\rangle$, where σ_i are Pauli