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High precision ray tracing in cylindrically symmetric electrostatics

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ABSTRACT

With the recent availability of a high order FDM solution to the curved boundary value problem, it is now possible to determine potentials in such geometries with considerably greater accuracy than had been available with the FDM method. In order for the algorithms used in the accurate potential calculations to be useful in ray tracing, an integration of those algorithms needs to be placed into the ray trace process itself. The object of this paper is to incorporate these algorithms into a solution of the equations of motion of the ray and, having done this, to demonstrate its efficacy. The algorithm incorporation has been accomplished by using power series techniques and the solution constructed has been tested by tracing the medial ray through concentric sphere geometries. The testing has indicated that precisions of ray calculations of 10^{-20} are now possible. This solution offers a considerable extension to the ray tracing accuracy over the current state of art.

1. Introduction

1.1. Elementary considerations

A particle moving within an electrostatic geometry experiences a force at each point of its movement proportional to the electric field at that point. Its path is governed by equations of motion which must include an accurate representation of this field. In the process described below the solution to the equations of motion will be found within the context of the finite difference method (FDM) and incorporate representations of the field from the potential algorithms themselves.

1.2. Finite difference method

Descriptions of the finite difference method (FDM) can be found in many references, the most succinct being that of Heddle [1]. Briefly it consists in placing a rectangular mesh over the geometry and then relaxing this mesh using an algorithmic process. A long standing problem with FDM has been its inability to incorporate curved boundaries in any but the lowest order manner. This difficulty has been recently been overcome and a solution has recently been found [2,3], with the result that accurate potentials can now be obtained for these curved boundary geometries. The determination of the accurate potential distributions has necessitated the creation of high order algorithms. In order to improve the accuracy of the ray trace solution, these algorithms are incorporated into it by the multiple use of power series techniques.

1.3. The solution for the coefficients c_j in the power series expansion of $v(r, z)$

The expansion of $v(r, z)$ as a truncated power series in r, z may be written:

$$v(r,\quad z) = c_0 + (c_1 \times z) + (c_2 \times r) + \left(c_3 \times z^2\right) + (c_4 \times z \times r) + \left(c_5 \times r^2\right) + \cdots + c_{s-1} r^n \tag{1}$$

where $c_0 \ldots c_{s-1}$ are coefficients of the expansion and n the expansion order, order being defined as the degree of the highest degree term in the expansion. It is noted that the expansion is made about a mesh point in the FDM net which in our representation of FDM occurs at integral values, while the point (r, z) is any point in the vicinity of this mesh point relative to that mesh point.

The determination of the coefficients c_j has been reported [4,5] and hence is very briefly sketched below. It is noted that the solution for the coefficients of any given order is called the algorithm for that order. As the underlying problem is cylindrically symmetric electrostatics, $v(r, z)$ must satisfy Laplace's equation at any point r, z within the geometry by

$$\left(\frac{(r+a)\,\partial^2}{\partial r^2} + \frac{\partial}{\partial r} + \frac{(r+a)\,\partial^2}{\partial z^2}\right) v(r,\quad z) = 0 \tag{2}$$

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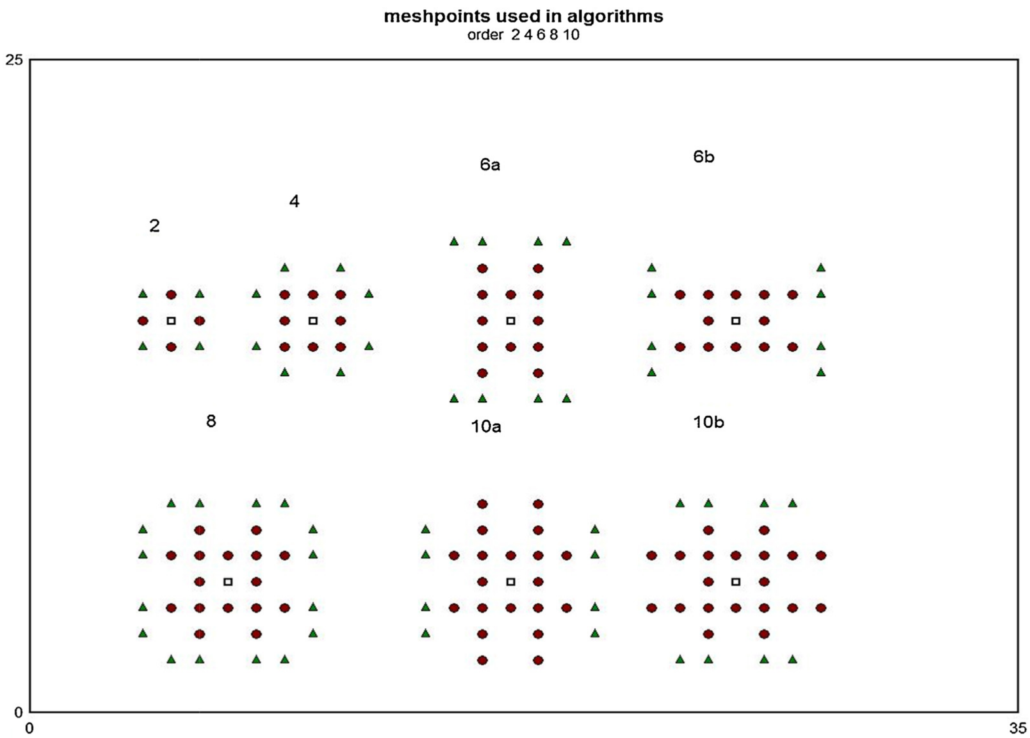

Fig. 1. The meshpoints used in the formulation of the various algorithms are depicted. The algorithm for a given order are formed by the average of all possible algorithms for that order (see text). It is noted that the very symmetric array of meshpoints used in the average algorithm for orders 6, 8, and 10 are the same as those of the order 8 algorithm, evincing considerable symmetry.

where r, z are the coordinates of any point in the geometry with reference to its closest mesh point and a the distance of that mesh point from the axis. This yields an equation in r, z which must be true for any r, z in the neighborhood of this meshpoint. The latter condition requires that in the resultant equation the coefficient of the term containing r^m, z^l must be zero, hence generating k equations from this single equation. As there are s coefficients to be determined an additional s–k equations must be found in order to have a complete set of s equations and s unknowns. These additional s–k equations are produced by evaluating $v(r, \quad z)$ at a selection of s–k meshpoints surrounding the meshpoint about which the expansion is made. The number of the additional meshpoints are a strong function of the order of the power series expansion. The solutions for order 2, 4, 6, 8, and 10 has been given in [2] and are used in the ray trace calculation below.

Fig. 1 shows the mesh points required for the various order algorithms. In this figure the points of any algorithm are given by the red discs together with one additional triangle point. The algorithm for that order is formed by averaging all possible algorithms of that order. It is further noted for order 6 and 10 the base algorithm consists of two types (a, b) but again the algorithm for those orders is the average of all possible (a and b) algorithms. For a further discussion see Ref. [2].

2. The ray tracing problem

2.1. The equations of motion and their solution

Consider a particle of charge q and mass m at a point r, z within a cylindrical geometry over which a rectangular mesh has been overlaid. It is assumed that the particle is at a point of its trajectory and both its position and velocity are known. The point itself is not necessarily at a mesh point location but is in the vicinity of the mesh point nearest to it about which the potential expansions described above are made. The solution for the particles subsequent motion is as follows. The equation of motion of this particle is determined by Newton's Law,

$$\mathbf{a} = \frac{q \times \mathbf{E}}{m} \tag{3}$$

where $\mathbf{E} = -\nabla v(r, \quad z)$, $\mathbf{a}$ its acceleration, and $v(r, \quad z)$ the potential at r, z.

The equations of motion of the particle follow immediately and after a straightforward but lengthy calculation are put into dimensionless form resulting in the following two equations:

$$\frac{\mathrm{d}^2 r(t)}{\mathrm{d}t^2} + \left(\text{beta} \times \frac{\partial v(r(t), \quad z(t))}{\partial r}\right) = 0 \tag{4}$$

$$\frac{\mathrm{d}^2 z(t)}{\mathrm{d}t^2} + \left(\text{beta} \times \frac{\partial v(r(t), \quad z(t))}{\partial z}\right) = 0 \tag{5}$$

where r, z, t, v, and beta are dimensionless while beta is given by

$$\text{beta} = \frac{q \times \left(\text{volt}/m\right)}{\left(\text{deltar}/\text{deltat}\right)^2} \tag{6}$$

deltar being the distance (in cm) between physical meshpoints and deltat the time (in seconds) for a particle with an initial kinetic energy E_0 (eV) to traverse a distance of one mesh spacing. To proceed $r(t)$ and $z(t)$are expanded as power series in t in which terms higher than degree n are neglected. The order of the expansion is defined as n, consistent with its use above.

$$r(t) = d_0 + d_1 t + d_2 t^2 + \cdots + dnt^n \tag{7}$$

$$z(t) = e_0 + e_1 t + e_2 t^2 + \cdots + e_n t^n \tag{8}$$

where $d_0, \ldots, d_n, e_0, \ldots, e_n$ being constants of the expansion. As previously stated, at the start of the time step (i.e., $t = 0$) the position