Journal of Electron Spectroscopy and Related Phenomena xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Journal of Electron Spectroscopy and Related Phenomena

journal homepage: www.elsevier.com/locate/elspec

Signal-to-noise ratio analysis of X-ray grating interferometry with the reverse projection extraction method

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ARTICLE INFO

Article history:
Available online xxx

Keywords:
Grating-based X-ray phase contrast imaging
Photon statistics
Mechanical errors
Signal-to-noise ratio

ABSTRACT

In the past decade, grating-based X-ray phase contrast imaging has attracted increasing attention. Particular efforts have been devoted to develop new information extraction method for its forthcoming practical applications. Here we discuss the noise properties of a new acquisition protocol, dubbed the "reverse projection" (RP) method, using the error propagation formula. We present a quantitative analysis of the signal-to-noise ratio of X-ray grating interferometry with this new method. As the major sources of noise, the contributions from photon statistics and mechanical errors are discussed in detail. The results show how the system parameters impact on the extracted absorption and refraction images and how they can be used to optimize the system design for foreseen practical applications, such as biomedical imaging and materials science.

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1. Introduction

Since the demonstration of X-ray grating interferometry by David et al. [1] and Momose et al. [2], grating-based X-ray phase contrast imaging (GBPCI) has stimulated interest of researchers of medical imaging [3–7]. In 2005, Weitkamp et al. discussed how to perform X-ray phase contrast imaging with a grating interferometer and other related technical issues [8]. Later, more practical Talbot-lau interferometry [9,10] was introduced into X-ray phase contrast imaging. In 2008, Pfeifer et al. proposed the concept of dark-field in X-ray grating interferometer [11]. More recently, many researchers focused on the extraction methods, which are extremely important for limiting the radiation dose. In 2010, the reverse projection (RP) method was introduced as a low dose, simple and fast method by Zhu et al. [12]. Later, Zanette et al. proposed the interlaced phase stepping and the sliding window phase stepping to reduce the radiation dose in phase tomography [13,14].

The noise property, an important figure of merit in imaging system, has been investigated for the phase stepping method by many groups [15–17]. In this contribution, we present a quantitative analysis of signal-to-noise ratios (SNRs) of images produced by the RP method. As the two major noise sources, the contributions from photon fluctuations and mechanical errors are analyzed in detail by using the error propagation formula [18]. Analytical expressions of the SNRs of images obtained by the RP method are derived, which demonstrates how system parameters affect the SNRs of the retrieved images using the RP method. The results provide useful guidelines to optimize the system parameters for practical applications.

2. Principle of the reverse projection method

First, let us review briefly the principle of the RP method. In an X-ray grating interferometer, the recorded photon number by the image detector can be written as [12]

$$N(x, y) = N_0(x, y) \exp\left[-\int \mu(x, y, z) dz\right] S\left[\frac{x_g}{z_N} + \theta(x, y)\right] \quad (1)$$

where N_0 is the incident total photon number, $\mu(x, y, z)$ the object's linear absorption coefficient, S the shifting curve (SC), x_g the relative displacement between the self-image of the $\pi/2$ phase grating

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http://dx.doi.org/10.1016/j.elspec.2014.04.011
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G1 and the analyzer grating G2, $z_N = Np_1^2/2\lambda$ denotes the Nth fractional Talbot distance with the grating pitch p_1, and the refraction angle $\theta(x, y)$ is related to the derivative of the phase shift $\Phi(x, y)$ by

$$\theta(x, y) = \frac{\lambda}{2\pi}\frac{\partial\Phi(x, y)}{\partial x} = -\int \frac{\partial\delta(x, y, z)}{\partial x}dz \quad (2)$$

with λ being the X-ray wavelength and δ the decrement of the real part of the object's refractive index.

Because the derivative of the shifting curve is fairly constant at its half slopes, the SC can be approximated as a linear function around. In other words, when small value θ satisfies $|\theta| \le p_2/4z_N$, $S(x_g/z_N + \theta)$ can be expressed by a first-order Taylor expansion at its half slopes

$$S(x_g/z_N + \theta) = S(x_g/z_N) + \frac{dS(x_g/z_N)}{d\theta}\theta = S(x_g/z_N)(1 + C\theta) \quad (3)$$

with $C = [dS(x_g/z_N)/d\theta]/S(x_g/z_N)$.

In the RP method, mutual conjugate images are collected with the sample rotated by 180°. Taking advantage of the fact that the refraction direction is inverted after rotation while the absorption remains the same, the photon numbers detected at the view angles ϕ and its corresponding reverse $\phi+\pi$ can be written as follows

$$N_\phi(x, y) = N_0(x, y, \phi)\exp\left[-\int \mu(x, y, z)dz\right] S(x_g)[1 + C\theta(x, y, \phi)] \quad (4)$$

$$N_{\phi+\pi}(-x, y) = N_0(x, y, \phi)\exp\left[-\int \mu(x, y, z)dz\right] S(x_g) [1 - C\theta(x, y, \phi)] \quad (5)$$

The absorption and refraction images can be obtained by solving the two equations above

$$M(x, y, z) = \int \mu(x, y, z)dz = \ln\left[\frac{2N_0S(x_g)}{N_\phi(x, y) + N_{\phi+\pi}(-x, y)}\right] \quad (6)$$

$$\theta(x, y, \phi) = \frac{1}{C}\frac{N_\phi(x, y) - N_{\phi+\pi}(-x, y)}{N_\phi(x, y) + N_{\phi+\pi}(-x, y)} \quad (7)$$

Notice that the incident total photon number is constant under parallel beam illumination, so the index of N_0 has been ignored in Eq. (6). Then, the three-dimensional distribution of the linear absorption coefficient as well as the decrement part of the refractive index can be obtained respectively by the Shepp–Logan (SL) and the Hilbert filters [19].

3. Noise analysis

In an imaging system, photon statistical noise and mechanical errors are the main factors affecting the image quality, which are discussed in this section. For sake of simplicity, next we will drop the indexes of the above physical quantities. Assume that $\bar{X}$ is the mean value of random variable X, $\sigma_{X,st}$ and $\sigma_{X,me}$ are respectively its standard deviations due to photon statistical noise and mechanical errors.

3.1. Photon statistical noise

The noise of the recorded projection images due to photon statistics translates into uncertainties of the absorption and refraction images. Using the error propagation formula [18], we can show the transfer procedure as follows

$$\begin{aligned}\sigma_{M,st}^2 &= \left(\frac{\partial M}{\partial N_\phi}\right)^2_{N_\phi=\bar{N}_\phi}\sigma_{N_\phi,st}^2 + \left(\frac{\partial M}{\partial N_{\phi+\pi}}\right)^2_{N_{\phi+\pi}=\bar{N}_{\phi+\pi}}\sigma_{N_{\phi+\pi},st}^2 \\ &= \left\{-\frac{\bar{N}_\phi + \bar{N}_{\phi+\pi}}{2N_0S}\frac{2N_0S}{[\bar{N}_\phi + \bar{N}_{\phi+\pi}]^2}\right\}^2\sigma_{N_\phi,st}^2 \\ &\quad + \left\{-\frac{\bar{N}_\phi + \bar{N}_{\phi+\pi}}{2N_0S}\frac{2N_0S}{[\bar{N}_\phi + \bar{N}_{\phi+\pi}]^2}\right\}^2\sigma_{N_{\phi+\pi},st}^2 \overset{Poisson}{=} \frac{1}{\bar{N}_\phi + \bar{N}_{\phi+\pi}} \\ &= \frac{1}{2N_0S\exp(-\bar{M})}\end{aligned} \quad (8)$$

and

$$\begin{aligned}\sigma_{\theta,st}^2 &= \left(\frac{\partial\theta}{\partial N_\phi}\right)^2_{N_\phi=\bar{N}_\phi}\sigma_{N_\phi,st}^2 + \left(\frac{\partial\theta}{\partial N_{\phi+\pi}}\right)^2_{N_{\phi+\pi}=\bar{N}_{\phi+\pi}}\sigma_{N_{\phi+\pi},st}^2 \\ &= \left[\frac{2\bar{N}_{\phi+\pi}}{C(\bar{N}_\phi + \bar{N}_{\phi+\pi})^2}\right]^2\sigma_{N_\phi,st}^2 + \left[\frac{-2\bar{N}_\phi}{C(\bar{N}_\phi + \bar{N}_{\phi+\pi})^2}\right]^2 \\ &\quad \sigma_{N_{\phi+\pi},st}^2 \overset{Poisson}{=} \frac{4\bar{N}_\phi\bar{N}_{\phi+\pi}}{C^2(\bar{N}_\phi + \bar{N}_{\phi+\pi})^3} = \frac{1}{2N_0S\exp(-\bar{M})}\left\{\frac{1}{C^2} - \bar{\theta}^2\right\}\end{aligned} \quad (9)$$

where we considered that the intensity of the recorded image follows the Poisson distribution, i.e. the variance equals its mean value. Only taking into account the photon statistics, the signal-to-noise ratios (SNRs) of the absorption and refraction images are given respectively by

$$\text{SNR}_{M,st} = \frac{\bar{M}}{\sigma_{M,st}} = \bar{M}\sqrt{2N_0S}\exp\left(\frac{-\bar{M}}{2}\right) = \sqrt{N_0}\bar{M}\ \exp\left(\frac{-\bar{M}}{2}\right) \quad (10)$$

$$\text{SNR}_{\theta,st} = \frac{\bar{\theta}}{\sigma_{\theta,st}} = \sqrt{N_0}\exp\left(\frac{-\bar{M}}{2}\right)\frac{C\bar{\theta}}{\sqrt{1 - C^2\bar{\theta}^2}} \quad (11)$$

where we considered $S = 0.5$ at its half slopes after its normalization.

We underline here that the SNRs of both absorption and refraction images are proportional to the square root of the incident photon number N_0 and related to the object's absorption property. Furthermore, for the absorption image the SNR is independent of the refraction angle $\bar{\theta}$, while the SNR of the refraction image is dependent on both $\bar{\theta}$ and $\bar{M}$. In addition, the SNR of the refraction image is relevant to the constant C. The slope $dS(x_g/z_N)/dx_g$ decreases with increasing z_N owing to spatial and temporal coherence of the X-ray beam [20,21], as a consequence, there must be an optimal distance z_N to maximize the SNR of the refraction image.

3.2. Mechanical errors

In the RP method, the analyzer grating G2 is fixed at one of the half slope positions. However, due to mechanical errors, the actual position around the ideal position can be described by a normal

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