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Nonlinear optical properties of a three-electron quantum dot with account of the Rashba spin–orbit interaction

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ABSTRACT

In this study, a detailed investigation of the nonlinear optical properties such as optical absorption and refractive index change associated with intersubband transitions in a three-electron quantum dot in two dimensions in the presence of the Rashba spin–orbit interaction has been carried out. We present the exact wave functions and energy levels of the system. Numerical results on typical GaAs/AlGaAs materials show that the decrease of the quantum dot radius blueshifts and amplifies the absorption coefficients as well as the refractive index changes, as expected. Additionally, an increase of the optical intensity and relaxation time considerably changes the absorption coefficients and the refractive index changes.

1. Introduction

Studies of discrete energy levels for three-electron quantum dots represent a particular interest because well-known model-independent methods of Faddeev equations and the HF can be applied for the description of these systems. Just like many other branches of physics, again the choice of the potential plays a crucial role. The three-electron quantum dots in different potentials including the harmonic, coulomb and Gaussian terms were studied in Refs. [1–8] using different methodologies. In such systems, the effect of spin–orbit interaction in the spectrum is definitely important. In technical words, we frequently investigate the Dresselhaus and Rashba interactions, which are respectively due to the electric field produced by the bulk inversion asymmetry of the material and the structural asymmetry of the heterostructure. There is now no doubt about the dependence of optical and electrical properties of confined electrons in quantum dots, wells and wires on the Rashba spin–orbit coupling and the experimental data does verify this claim [9–17].

From the other side, it is well known that the nonlinear optical properties can be enhanced dramatically in low-dimensional quantum systems. These strong nonlinear optical properties, such as harmonic generation, refractive index changes, optical rectification, optical absorption effects and so on, have a great potential in several device applications, such as in far infrared laser amplifiers and waveguide splitters. Hence, optical properties of QDs have been investigated both experimentally and theoretically by many authors [18–30]. On the other hand, although the nonlinear optical properties of excitons in QDs are examined widely in the literature, those of electrons have just been investigated only for the past few years [29–32]. Because the energy eigenvalues and their wave functions related to single electron can be simply determined analytically. In more than one-electron systems, determining the energy eigenvalues and relevant wave functions are not so simple because of the Coulomb interaction between the electrons. In this work, we will devote our calculations to the nonlinear optical absorption coefficients as well as the refractive index changes of a three-electron QD by considering Columbic and Rashba spin–orbit interaction. The electronic states and energy eigenvalues are exactly solved. To the best of our knowledge, this problem has not been studied extensively in the literature.

2. Model and theory

The Hamiltonian H_{tot} of the three-electron-quantum dot in two dimensions with the Coulomb potential and by considering Rashba spin–orbit interaction is

$$H_{tot} = \sum_{j,l=1,j<l}^{3} \left(\frac{p_j^2}{2m_e^*} + \frac{e^2}{\kappa|\vec{r_j}-\vec{r_l}|} + H_{S-O}^{(j)} \right) + V_{Con} \tag{1}$$

where the Rashba spin–orbit interaction, the confinement potential and the standard Pauli matrices are

$$V_{Con} = \begin{cases} 0, & \rho < \rho_0 \\ V_0, & \rho > \rho_0 \end{cases} \tag{2}$$

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$$H_{S-O}^{(j)}=\alpha_R(\sigma_x^{(j)}p_y^{(j)}-\sigma_y^{(j)}p_x^{(j)}) \tag{3}$$

$$\sigma_x=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \sigma_y=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \tag{4}$$

by introducing Jacobi coordinate as

$$\vec{R}=\frac{\vec{r_1}+\vec{r_2}+\vec{r_3}}{3},\quad \vec{\rho}=\frac{\vec{r_1}-\vec{r_2}}{\sqrt{2}},\quad \vec{\lambda}=\frac{\vec{r_1}+\vec{r_2}-2\vec{r_3}}{\sqrt{6}} \tag{5}$$

By using the hyperspherical coordinates including the hyperradius x and the hyperangle ξ respectively defined via [5]:

$$x=\sqrt{\rho^2+\lambda^2},\quad \xi=\arctan\left(\frac{\rho}{\lambda}\right) \tag{6}$$

the total Hamiltonian can be written in the form

$$H_{tot}=\frac{p_{c.m.}^2}{2M_e^*}+\frac{\hbar\alpha_R}{i}\begin{pmatrix}0 & i\frac{\partial}{\partial R_x}+\frac{\partial}{\partial R_y}\\ -i\frac{\partial}{\partial R_x}+\frac{\partial}{\partial R_y} & 0\end{pmatrix}+V_{Con}+\frac{p_x^2}{2m_e^*}+\frac{c}{x} \tag{7}$$

where $M_e^*=3m_e^*$

$$c=\frac{e^2}{\kappa}\left(\frac{\sqrt{1+\alpha^2}}{\sqrt{2}\alpha}+\frac{\sqrt{2(1+\alpha^2)}}{\sqrt{3}+\alpha}+\frac{\sqrt{2(1+\alpha^2)}}{\sqrt{3}-\alpha}\right) \tag{8}$$

where $\alpha=|\vec{\lambda}|/|\vec{\rho}|$. The total Hamiltonian H_{tot} is separated in terms of the center of mass and relative coordinate parts, i.e.

$$H_{tot}=H(R_{c.m.})+H(x) \tag{9}$$

where $H(R_{c.m.})$ content V_{Con}. An equivalent statement is that the eigenvalue equation of the system is

$$E_{m,n,\gamma}=E_{m.c.m.}+E_{n,\gamma} \tag{10}$$

where m, n and γ are the quantum numbers. Therefore, we represent the wave functions by Considering total Hamiltonian, the

$$\Psi_{n,\gamma,m}(x,R_{c.m.})=Q_{n,\gamma}(x)\psi_m(R_{c.m.}) \tag{11}$$

Schrodinger equation for the hyperradius is

$$\left\{\frac{-\hbar^2}{2m_e^*}\left[\frac{d^2}{dx^2}+\frac{3d}{xdx}-\frac{\gamma(\gamma+2)}{x^2}\right]+\frac{c}{x}\right\}Q_{n,\gamma}(x)=E_{n,\gamma}Q_{n,\gamma}(x) \tag{12}$$

The eigenvalues of above equation is

$$E_{n,\gamma}=\frac{-m_e^*c^2}{2\left(n+\gamma+\frac{3}{2}\right)^2}=-\frac{m_e^*\left\{\frac{e^2}{k}\left[\frac{\sqrt{1+\alpha^2}}{\sqrt{2}\alpha}+\frac{\sqrt{2(1+\alpha^{2)}}}{\sqrt{3}+\alpha}+\frac{\sqrt{2(1+\alpha^2}}{\sqrt{3}-\alpha}\right]\right\}^2}{2\left(n+\gamma+\frac{3}{2}\right)^2} \tag{13}$$

and the eigenfunction can be written in the form

$$Q_{n,\gamma}(x)=N_{n,\gamma}x^{\gamma}e^{-(1/h)\sqrt{-2m_e^*}x}L_n^{2\gamma+2}\left(\frac{2}{\hbar}\sqrt{-2m_e^*E_{n,\gamma}}x\right) \tag{14}$$

where $N_{n,\gamma}$ is the normalization constant and $L_k^n(x)$ is the Laguerre polynomial. Schrodinger equation appears for $H(R_{c.m.})$ as

$$\left[\frac{-\hbar^2}{2M_e^*}\left(\frac{\partial^2}{\partial R_x^2}+\frac{\partial^2}{\partial R_y^2}\right)+\frac{\hbar\alpha_R}{i}\begin{pmatrix}0 & i\frac{\partial}{\partial R_x}+\frac{\partial}{\partial R_y}\\ -i\frac{\partial}{\partial R_x}+\frac{\partial}{\partial R_y} & 0\end{pmatrix}\right]\psi_{1,m,c.m.}(R_{c.m.})=E_{1,m,c.m.}\psi_{1,m,c.m.}(R_{c.m.}) \tag{15}$$

where

$$\psi_{m,c.m.}(R_{c.m.})=\begin{pmatrix}u(R_{c.m.})e^{im\varphi}\\ w(R_{c.m.})e^{i(m+1)\varphi}\end{pmatrix} \tag{16}$$

By introducing the dimensionless parameters according to

$$\epsilon_m=\frac{2M_e^*R_{0,c.m.}^2E_{m,c.m.}}{\hbar^2},\quad \alpha=\frac{2M_e^*R_{0,c.m.}\alpha_R}{\hbar},\quad R=\frac{R_{c.m.}}{R_{0,c.m.}} \tag{17}$$

The function $u(m,\varepsilon_m,\alpha,R)$ and $w(m+1,\varepsilon_m,\alpha,R)$ are as follows:

$$u(m,\varepsilon_m,\alpha,R)=B\left(-\frac{J_{m+1}(k^-)+J_{m+1}(k^+)}{J_{m+1}(k^-)-J_{m+1}(k^+)}[J_m(Rk^-)+J_m(Rk^+)]+[J_m(Rk^-)-J_m(Rk^+)]\right) \tag{18}$$

$$w(m+1,\varepsilon_{m+1},\alpha,R)=B\left(-\frac{J_{m+1}(k^-)+J_{m+1}(k^+)}{J_{m+1}(k^-)-J_{m+1}(k^+)}[J_{m+1}(Rk^-)-J_{m+1}(Rk^+)]+[J_{m+1}(Rk^-)+J_{m+1}(Rk^+)]\right) \tag{19}$$

where

$$k_1^{\pm}=\sqrt{\epsilon_m+\frac{\alpha^2}{4}}\pm\frac{\alpha}{2} \tag{20}$$

And the energy eigenvalues $E_{m,c.m.}$ of the system are found from the following equation:

$$[J_{m+1}(k^-)-J_{m+1}(k^+)][J_m(k^-)-J_m(k^+)]-[J_m(k^-)+J_m(k^+)][J_{m+1}(k^-)+J_{m+1}(k^+)]=0 \tag{21}$$

We calculate the absorption coefficients and refractive index changes using the linear $\chi^{(1)}$ and third-order $\chi^{(3)}$ optical susceptibilities. The analytical forms of $\chi^{(1)}$ and $\chi^{(3)}$ are obtained from modeling a QD as a two-level system. The susceptibility χ is related to the absorption coefficient $\alpha(v)$ by

$$\alpha(v)=v\sqrt{\frac{\mu_0}{\varepsilon_r}}Im[\varepsilon_0\chi(v)] \tag{22}$$

where μ_0 is the permeability of the system. $\varepsilon_r=n_r^2\varepsilon_0$ is the real part of the permittivity, n_r is the medium refractive index and ε_0 is the permittivity of vacuum. χ is the Fourier component of $\chi(t)$ with $\exp(-i\omega t)$ dependence. Using the compact density-matrix method, the optical absorption coefficient is given by [31]

$$\alpha(v)=\alpha^{(1)}(v)+\alpha^{(3)}(v,I) \tag{23}$$

where

$$\alpha^{(1)}(v)=\frac{4\pi\beta_{FS}\sigma_s}{n_re^2}hv|M_{fi}|^2\delta(E_{fi}-hv) \tag{24}$$

and

$$\alpha^{(3)}(v,I)=-\frac{32\pi^2\beta_{FS}^2\sigma_sI}{n_r^2e^4\hbar\Gamma_{ff}}hv|M_{fi}|^4\delta(E_{fi}-hv)^2\times\left\{1-\frac{|M_{ff}-M_{ii}|^2}{}\times4|M_{fi}|^2\frac{[(hv-E_{fi})^2-(\hbar\Gamma_{fi})^2+2E_{fi}(E_{fi}-hv)]}{E_{fi}^2+(\hbar\Gamma_{fi})^2}\right\} \tag{25}$$

are the linear and the third-order nonlinear optical absorption coefficients, respectively. e is the electronic charge of an electron, σ_s is the electron density in the QD, $\beta_{FS}=e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant, I is the incident optical intensity, and hv is the photon energy. The δ function in Eqs. (3) and (4) are replaced by a narrow Lorentzian by means of

$$\delta(E_{fi}-hv)=\frac{\hbar\Gamma_{fi}}{\pi[(hv-E_{fi})^2+(\hbar\Gamma_{fi})^2]} \tag{26}$$

Here Γ is the phenomenological operator. Nondiagonal matrix element $\Gamma_{fi}(f\neq i)$ of operator Γ, which is called as the relaxation rate of fth state and ith state, is the inverse of the relaxation time