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Visualization and analysis of modulated pulses in magnetic resonance by joint time–frequency representations

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ABSTRACT

We study the utility of joint time–frequency representations for the analysis of shaped or composite pulses for magnetic resonance. Such spectrograms are commonly used for the visualization of shaped laser pulses in optical spectroscopy. This intuitive representation provides additional insight compared to conventional approaches, which exclusively show either temporal or spectral information. We focus on the short-time Fourier transform, which provides not only amplitude but also phase information. The approach is illustrated for broadband inversion pulses, multiple quantum excitation and broadband heteronuclear decoupling. The physical interpretation and validity of the approach is discussed.

1. Introduction

In this paper we use a powerful approach to represent highly modulated NMR pulse shapes, which helps to understand their underlying mode of action. Conventionally, pulse shapes are represented in the time domain, i.e. the pulse components $u_x(t)$ and $u_y(t)$ (or alternatively the pulse amplitude and phase) are shown as a function of time. However, this representation often provides little (if any) intuition about the mode of action of a given pulse. In particular, numerically optimized pulses often look like random noise and are notoriously difficult to interpret [1–10]. As demonstrated in the following, the underlying structure of the optimized pulse shapes can be revealed by a joint time–frequency representation, which is commonly used in many fields such as the analysis of acoustic signals, seismic data and shaped laser pulses [11–15] but its potential has hitherto not been exploited in magnetic resonance applications. The concept of joint time–frequency representations is best illustrated by musical scores, which represent the temporal sequence of individual notes or chords. Different families of algorithms exist for the generation of time–frequency distributions from time-domain data, such as the Wigner-Ville approach [16–23], short-time Fourier transform (STFT, spectrogram representation) [11,12], wavelet transforms [21,24–27] and the von Neumann representation [15,28].

In this paper, the STFT technique is used to represent not only the temporally and spectrally resolved pulse amplitude but also the corresponding pulse phase. The STFT corresponds to a transformation of a given pulse shape to a local rotating frame for each offset frequency and an additional smoothing based on a gate function, creating a weighted moving average of the pulse in the local frame of reference. The degree of smoothing depends on the width of the gate function. The error in spin dynamics introduced by the pulse smoothing is analysed in terms of a quality factor based on Average Hamiltonian Theory [29,30].

2. Theory

2.1. Radio-frequency pulses

Radio-frequency (rf) pulses $s(t)$ can be described in the time domain as

$$s(t) = u_x(t) + iu_y(t), \quad (1)$$

where $s(t)$ is the complex pulse shape and $u_x(t)$ and $u_y(t)$ are the amplitudes of the x- and y-components of the radio frequency (or microwave frequency) field, or by

$$s(t) = A(t)e^{i\varphi(t)}, \quad (2)$$

where $A(t)$ and $\varphi(t)$ are the temporal amplitude and phase (c.f. Fig. 1).

$$A(t) = \sqrt{u_x(t)^2 + u_y(t)^2}, \quad (3)$$

$$\varphi(t) = \arctan\left(\frac{u_y(t)}{u_x(t)}\right). \quad (4)$$

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The Fourier transform converts the pulse from the time domain $s(t)$ to the frequency domain by

$$S(\nu) = \int_{-\infty}^{\infty} s(t) e^{-i2\pi\nu t}\, dt, \tag{5}$$

where $S(\nu)$ is the pulse spectrum

$$S(\nu) = A(\nu) e^{i\varphi(\nu)}, \tag{6}$$

defined by the spectral amplitude $A(\nu)$ and phase $\varphi(\nu)$ (c.f. Fig. 2). The one-dimensional pulse or spectrum representations do not provide temporal and spectral resolution simultaneously. However, the temporal–spectral couplings can be revealed by a two-dimensional joint time–frequency representation [31].

2.2. Short-time Fourier transform (spectrogram)

Here we focus on the short-time Fourier transform method to create a joint time–frequency representation of a given radio-frequency pulse [11–14]. In order to introduce temporal resolution in the spectral representation of the pulse, $s(t)$ can be segmented by a gate function $g(t)$ into intervals. The gate function is centered on t and suppresses the pulse outside the interval $[t - \Delta t,\ t + \Delta t]$. Different classes of functions can be used as gate function. Here we define the gate function $g(t)$ by a normalized Gaussian window (Gabor transform) [11,12]

$$g(\tau) = \begin{cases} \frac{1}{N} e^{-\frac{1}{2}\frac{\tau^2}{\sigma^2}} & \text{for } \tau \in [-\Delta t, \Delta t] \\ 0 & \text{else} \end{cases} \tag{7}$$

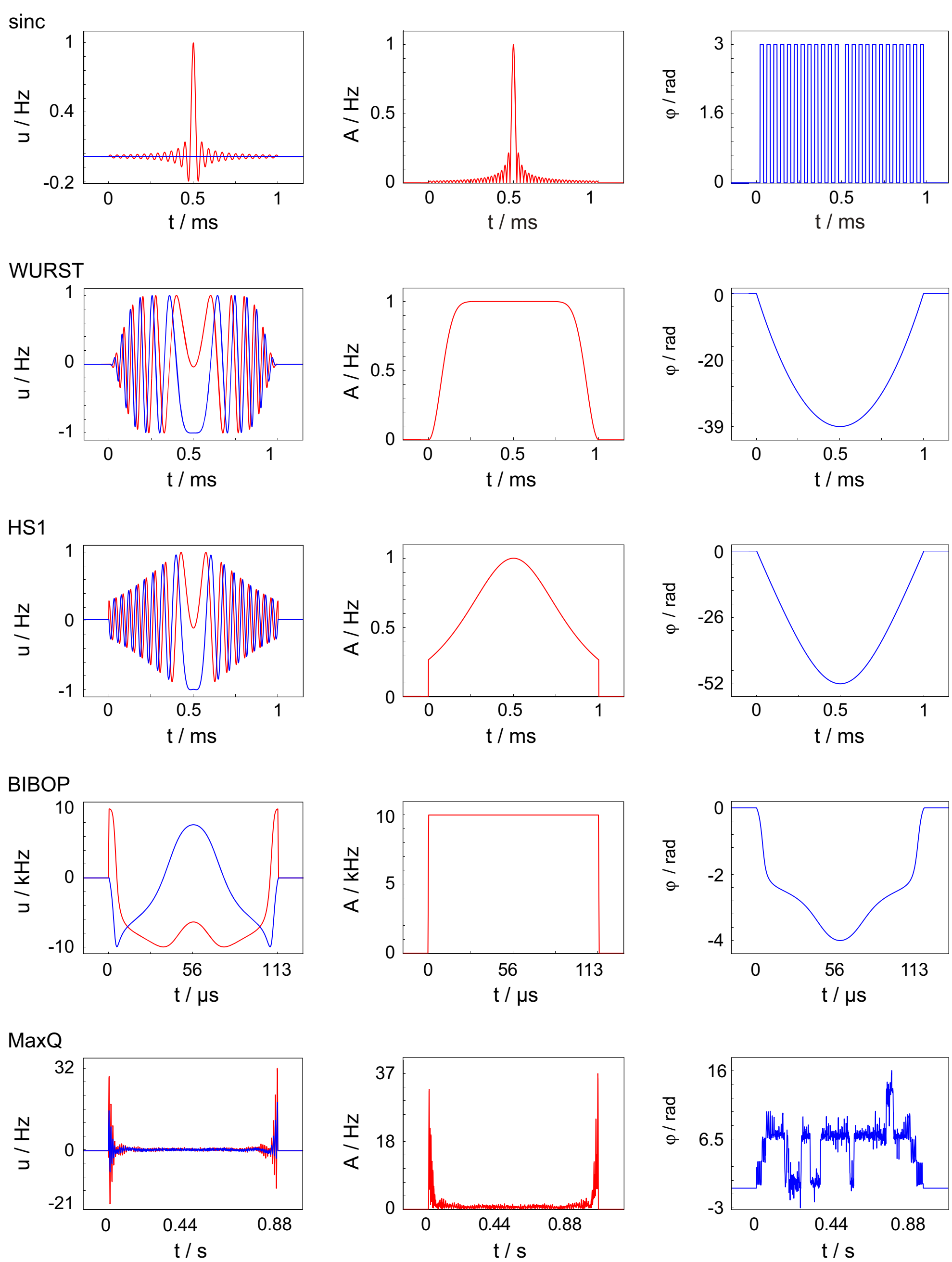

Fig. 1. Pulse shapes of example pulses (sinc, WURST, HS1, BIBOP, MaxQ) are represented in the temporal domain as amplitudes of the x- and y-components $u_x(t)$ (red) and $u_y(t)$ (blue) in the left column. The temporal amplitude $A(t)$ (red) and phase $\varphi(t)$ (blue) are depicted in the middle and right column, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)