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Magnetic field homogeneity perturbations in finite Halbach dipole magnets

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ABSTRACT

Halbach hollow cylinder dipole magnets of a low or relatively low aspect ratio attract considerable attention due to their applications, among others, in compact NMR and MRI systems for investigating small objects. However, a complete mathematical framework for the analysis of magnetic fields in these magnets has been developed only for their infinitely long precursors. In such a case the analysis is reduced to two-dimensions (2D). The paper details the analysis of the 3D magnetic field in the Halbach dipole cylinders of a finite length. The analysis is based on three equations in which the components of the magnetic flux density B_x, B_y and B_z are expanded to infinite power series of the radial coordinate r. The zeroth term in the series corresponds to a homogeneous magnetic field B_c, which is perturbed by the higher order terms due to a finite magnet length. This set of equations is supplemented with an equation for the field profile $B(z)$ along the magnet axis, presented for the first time. It is demonstrated that the geometrical factors in the coefficients of particular powers of r, defined by intricate integrals are the coefficients of the Taylor expansion of the homogeneity profile $(B(z) - B_c)/B_c$. As a consequence, the components of **B** can be easily calculated with an arbitrary accuracy. In order to describe perturbations of the field due to segmentation, two additional equations are borrowed from the 2D theory. It is shown that the 2D approach to the perturbations generated by the segmentation can be applied to the 3D Halbach structures unless r is not too close to the inner radius of the cylinder r_i. The mathematical framework presented in the paper was verified with great precision by computations of B by a highly accurate integration of the magnetostatic Coulomb law and utilized to analyze the inhomogeneity of the magnetic field in the magnet with the accuracy better than 1 ppm.

1. Introduction

Halbach dipole cylindrical magnets [1,2] are in essence hollow cylinders magnetized perpendicular to the axis $\hat{z}$ of the cylinder in such a way that the magnetization direction rotates continuously with its angular position by two space cycles, as it is shown in Fig. 1. If such magnet is infinitely long, it generates a perfectly uniform magnetic field confined to its bore and directed perpendicular to the cylindrical axis. All practical implementations of the theoretical infinite Halbach cylinders have of course finite length and the shorter are the magnets, the more their ends disturb the field homogeneity. As a consequence, the magnetic field ceases to be a pure dipole one. We will refer to such magnets as quasi dipoles.

The Halbach quasi dipole magnets have received considerable attention, since they are applied in compact NMR, MRI and EPR systems [3–8] as well as in magnetic filtration [9] and refrigeration systems [10,11]. They are also applied to produce biasing field in electronic filters, as general utility laboratory magnets and in wigglers and undulators for free electron lasers [12]. The largest, nearly 2 tons Halbach magnet is at the International Space Station (ISS) as a part of the alpha magnetic spectrometer [13] and the strongest one generates the magnetic field of the flux density 5 T [14]. The NMR and MRI Halbach magnets described in the literature have the inner radius and homogeneity area that range from a few mm to a few cm [3,6,15–19] and consequently, only small objects can be investigated by them. However, recently technology suitable for a compact MRI system was reported with the Halbach-type magnet large enough to accommodate the entire human body and sufficiently light to be transported together with the whole system to the ISS [20,21].

Although the Halbach quasi dipole cylinders have been utilized in practice for more than 30 years, a full analytical theory of the magnetic field they generate was developed only for those of a very high bore aspect ratio [1,2,22]. In such magnets, the end effects can be neglected and the magnets can be treated as pure dipoles. Since the lines of **B** are homogeneously spaced and perpendicular to the

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axis of the magnet the mathematical description of the field can be reduced to two dimensions (2D). Unlike the 2D theory of the infinite-length Halbach magnets, the mathematical framework for their three dimensional (3D) finite-length counterparts of the low aspect ratio was poorly developed. Zijlstra [23,24] derived an exact equation for the magnetic flux density in the center of such magnets. Ravaud and Lemarquand [25] presented an analytical theory of 3D magnetic fields in the Halbach cylinders, but the immensely complicated equations they derived by Coulomb's method contain elliptical integrals that do not present an analytical solution. An attempt to derive the field equations from the scalar potential was undertaken earlier by Ni Mhiocháin and co-workers [26], but they neglected some important terms, which will be shown later in this paper. Moreover, according to the Ni Mhiocháin's method, the geometrical factors at the powers of r are defined by cumbersome, though analytically computable integrals. Soltner and Blümler [5] used the dipole approximation to calculate within the accuracy of about 5000 ppm the field profile at the axis of a stack of very short Halbach-like structures. The shortage of an efficient mathematical framework for analytic computations of the field inhomogeneity is partially compensated by the availability of highly accurate programs solving semi-analytical field equations [25,27] or solving the differential field equations by the finite element or the boundary element method [28–31]. However, in order to compute the inhomogeneity of the field by these programs B must be numerically mapped in each computer model of the magnet to find extremes of B in the maps.

The field homogeneity in all real constructions is also disturbed by the segmentation of the cylinder into pieces uniformly magnetized in the directions consistent with the Halbach prescription. The stepwise change of the magnetization direction in the segmented magnets approximates the continuous rotation of the magnetization that due to technological constrains cannot be realized in practice. Halbach showed [2] that the segmentation in 2D perturbs the field homogeneity with harmonics of the order being a multiple of the number of the segments. These harmonics vanish in the magnet bore with the normalized r_o distance from the inner magnet surface.

We present a set of the analytically solvable equations for the 3D magnetic field in the bore of the Halbach quasi dipole cylinders of the finite length and utilize them to compute the field inhomogeneity in the central area of the magnet with the accuracy better than 1 ppm. Though such an accuracy of the computations requires computer support, some reasonable conclusions can be drawn from the theory using a pocket calculator. This set comprises the corrected version of the field equations for the Halbach cylinders originally derived by Ni Mhiocháin and co-workers [32]. The equations can be interpreted as representing a homogeneous magnetic field of B described by the zeroth term in the power series of the normalized distance from the magnet center r perturbed by the higher-order terms in this series. The coefficients of each power of r contain geometrical factors that are intricate integrals. We substantiated that these integrals are the coefficients of the Taylor series expansion of the field profile along the magnet axis $B(z)$, whose analytical equation is presented for the first time in this paper. We also showed that in an area which is not too close to the inner wall of the cylinder, the perturbations of the 3D field by segmentation can be described with a high accuracy by the equations derived by Halbach for the 2D field. All the analytical equations in this paper were verified as to be in very good agreement with the numerical results generated by a computer program based on a highly accurate integration of the magnetostatic Coulomb law (IMCL) [33].

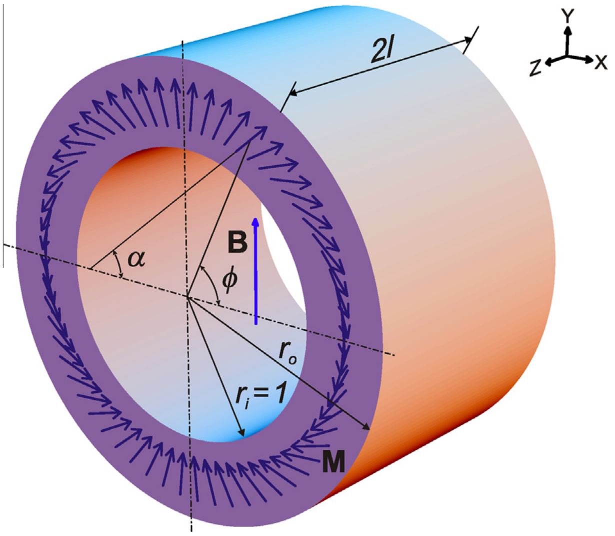

Fig. 1. Changes in the orientation α of the magnetization **M** with the azimuthal angle ϕ in a section of a Halbach dipole cylinder. The magnet is localized at the center of the coordinate system. All the linear dimensions are normalized by r_i.

2. Halbach dipole cylindrical magnets

The discovery in the seventies of the twentieth century of high-energy Sm–Co permanent magnets of an almost rectangular hysteresis loop stimulated development of insertion devices for accelerators based on these magnets [34]. This development was initiated by Halbach's research [1,2] on long hollow cylindrical structures magnetized perpendicularly to the magnet axis so that the magnetization orientation changes continuously or stepwise with the azimuthal angle by a definite number $N + 1$ of space cycles. Due to these changes a multipole field of the order $2N$ is generated in an elongated volume of the bore of the induction **B** perpendicular to the long, say $\hat{z}$, axis of this volume. The structures are considered to be infinite or long enough so that end effects can be neglected. In this case the magnetic field can be regarded then as two-dimensional.

For $N = 1$ and the continuous change of the magnetization orientation the Halbach cylinder produces a perfectly homogeneous dipole magnetic field. Hence, such a cylinder is called a dipole. In the dipole Halbach cylinders the relationships between azimuthal angle ϕ and magnetization direction α is

$$\alpha = (N+1)\phi + \alpha_0, \tag{1}$$

where α_0 is the orientation of the magnetization at $\phi = 0$. For a section of an infinite dipole Halbach cylinder ($N = 1$), shown in Fig. 1 $\alpha_0 = 3\pi/2$.

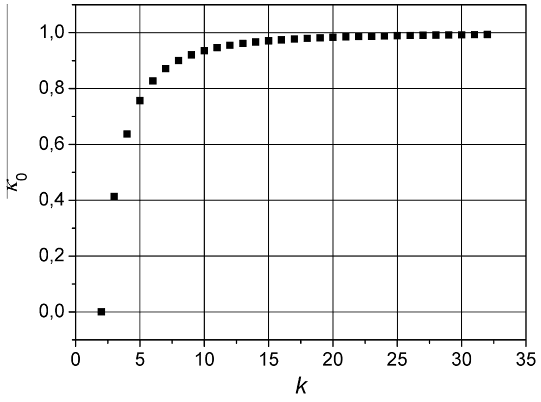

Fig. 2. Dependency of the coefficient κ_0 on k.