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Arbitrary magnetic field gradient waveform correction using an impulse response based pre-equalization technique

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ABSTRACT

The time-varying magnetic fields used in magnetic resonance applications result in the induction of eddy currents on conductive structures in the vicinity of both the sample under investigation and the gradient coils. These eddy currents typically result in undesired degradations of image quality for MRI applications. Their ubiquitous nature has resulted in the development of various approaches to characterize and minimize their impact on image quality.

This paper outlines a method that utilizes the magnetic field gradient waveform monitor method to directly measure the temporal evolution of the magnetic field gradient from a step-like input function and extracts the system impulse response. With the basic assumption that the gradient system is sufficiently linear and time invariant to permit system theory analysis, the impulse response is used to determine a pre-equalized (optimized) input waveform that provides a desired gradient response at the output of the system. An algorithm has been developed that calculates a pre-equalized waveform that may be accurately reproduced by the amplifier (is physically realizable) and accounts for system limitations including system bandwidth, amplifier slew rate capabilities, and noise inherent in the initial measurement. Significant improvements in magnetic field gradient waveform fidelity after pre-equalization have been realized and are summarized.

1. Introduction

The application of time-varying magnetic fields (pulsed field gradients) required for spatial localization in magnetic resonance (MR) applications (such as imaging, pulsed gradient spin echo (PGSE) [1], and gradient enhanced spectroscopy [2–5]) result in the induction of eddy currents on conductive structures within the vicinity of the sample under investigation, the gradient coils, and magnet structure. These eddy currents, a result of Faraday's law of electromagnetic induction, typically result in undesired degradations of the quality of the acquired data. The ubiquitous nature of eddy currents has resulted in the development of various approaches to characterize and minimize their impact on the acquired data. Many of these approaches have been developed to mitigate the eddy current effects in magnetic resonance imaging (MRI); some of which are outlined below.

One approach to minimize eddy current impacts is to use passive [6] and active [7–11] gradient shielding. The shields are used to minimize or eliminate the magnetic fields beyond the gradient coil which minimize the induction of eddy currents in the surrounding structure. However, one of the consequences of shielding is a reduction in the overall efficiency of the gradient coil [12]. Furthermore, this approach does not address eddy currents induced on the radiofrequency (RF) probe structure which is located inside the gradient coils.

A second approach (MRI specific) is to use post-processing techniques to correct the eddy current distorted *k*-space data. This can be accomplished using techniques such as gridding [13,14] where *k*-space that does not lie uniformly on a Cartesian plane is corrected based on *a priori* knowledge of the *k*-space trajectory [15–27].

The methods used to characterize the *k*-space trajectory [15–21] can also be used in a third approach to minimize eddy current impacts where the current delivered to the gradient coils is modified such that induced effects are minimized [12]. Typically, this is accomplished through the characterization of the decay of the magnetic field gradient following a gradient pulse. The decay is fit to multiple exponential functions with distinct time constants and amplitudes which ultimately determines the coefficients required to set and optimize the pre-emphasis [28,29] network. Exponential function based pre-emphasis is used in many MRI scanners today and many different methods for setting and optimizing their parameters have been developed [30–34]. These methods typically ensure [31,34] or assume [32,33] that the eddy

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currents associated with the leading and falling edge of a gradient pulse do not interact; therefore, the approach is not general. Furthermore, pre-emphasis is typically implemented using a number of exponential functions with varying amplitudes and time constants that are added to the desired gradient waveform either digitally or using discrete analog circuits. A shortcoming of this implementation is that an addition operation is employed in the waveform correction whereas a convolution-based operation is required. Another shortcoming of this approach is the implicit assumption that the field produced by the gradient coil is identical to that produced by the induced eddy currents.

Recently, gradient characterization was reported in [35] where a dynamic field camera [25] is used to determine the impulse response of the gradients through the application of triangular shaped input gradients. This method uses multiple probes to determine the spatiotemporal evolution of the magnetic field gradient and can be used to determine appropriate dynamic field shimming when combined with spherical harmonic basis functions.

This paper outlines our recently developed method that utilizes the magnetic field gradient waveform monitor (MFGM) [22,24] method to directly measure the temporal evolution of the magnetic field gradient and extracts the system impulse response. The impulse response then permits the calculation of an optimized input waveform that will provide a desired gradient response at the output. Note that techniques such as magnetic field monitoring (MFM) [25] could be applied with our method such that corrections of large scale gradient system effects and subtle field dynamics are concurrently addressed.

2. Theory

2.1. System impulse response

The fundamental assumption underlying the method is that the system is sufficiently linear and time-invariant (LTI) that system theory is applicable. This assumption permits the system to be characterized using the block diagram shown in Fig. 1.

A set of system operational constraints are required to ensure that the system is operated within a region such that LTI operation results. System theory indicates that the system output $y(t)$ is the convolution of the system input $x(t)$ and the impulse response $h(t)$ and is mathematically represented as

$$x(t) * h(t) = y(t) \quad (1)$$

The operator has experimental control over the system input $x(t)$ and the system output $y(t)$, the temporal evolution of the magnetic field gradient waveform, may be measured with the MFGM method. Therefore, we have the ability to determine the impulse response $h(t)$ through a deconvolution operation. Knowledge of the impulse response permits the application of system theory in order to determine an input waveform that results in a desired output waveform. The impulse response may be readily determined from the system response to a step function input. The derivation of this result is rarely shown although it is frequently quoted. The

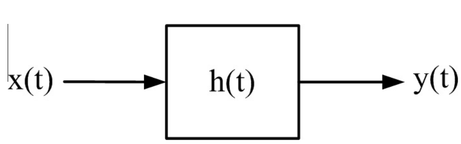

Fig. 1. Block diagram representation of the input magnetic field gradient waveform $x(t)$, the system impulse response $h(t)$, and the system output magnetic field gradient waveform $y(t)$ for a linear time invariant system.

derivation of the method used to determine the system impulse response from a step function input is provided below for completeness.

Dirac's delta function or impulse function, $\delta(t)$, is defined in [36] as an intense pulse with unit area and is expressed mathematically as

$$\delta(t) = 0 \quad t \neq 0 \quad (2)$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \quad (3)$$

This mathematically convenient notation is often interpreted by

$$\lim_{\tau \to 0} \int_{-\infty}^{\infty} \tau^{-1} \Pi\left(\frac{t}{\tau}\right) dt = 1 \quad (4)$$

where $\tau^{-1}\Pi(\frac{t}{\tau})$ is a boxcar (or rectangle) function of height τ^{-1} and width (or base) τ. Note that the limit of Eq. (4) is unity based on the definition of the impulse function in (3). A beneficial property of Dirac's delta function is that $\int_{-\infty}^{x} \delta(t)dt$ equals unity for $x > 0$ and it equals zero for $x \leq 0$ resulting in a definition of the unit step function, $u(t)$,

$$\int_{-\infty}^{x} \delta(t)dt = u(t) \quad (5)$$

Correspondingly, differentiating both sides of (5) yields

$$\frac{d}{dt} u(t) = \delta(t) \quad (6)$$

As previously stated, the system output $y(t)$ is the convolution of the input waveform $x(t)$ and the system impulse response $h(t)$ (refer to Eq. (1)). Substituting a step function $u(t)$ for the input function results in

$$u(t) * h(t) = y(t) \quad (7)$$

Differentiating (7) results in

$$\frac{d}{dt} y(t) = \frac{d}{dt}[u(t) * h(t)] = \delta(t) * h(t) \quad (8)$$

following the application of the derivative theorem in conjunction with the convolution theorem [36]. A function convolved with a delta function results in the original function along with any time-shift (if present) [37]. Therefore, Eq. (8) is equivalent to

$$\frac{d}{dt} y(t) = \delta(t) * h(t) = h(t) \quad (9)$$

The differentiation of a system response resulting from a step input yields the system impulse response. It is this equality that will be applied to extract the system impulse response in this work.

Since the application of an infinite step function is not feasible, the function that is used as the input is a boxcar function with sufficient duration such that the system achieves steady state during the measurement. Sufficient duration may be determined through observation of the temporal evolution of the magnetic field gradient waveform and ensuring that the system has achieved a steady state prior to applying the falling edge of the gradient pulse. The measured data can then be truncated prior to the effects of the falling edge of the gradient waveform is observed in order to approximate step response data. Differentiation of the measured step response data yields the desired system impulse response.

2.2. Pre-equalization

Knowledge of the system impulse response permits the calculation of the required input waveform that provides a desired output waveform. In this case, the input function, $x(t)$ in Eq. (1) is the