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Communication

Direct design of 2D RF pulses using matrix inversion

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ABSTRACT

Multi-dimensional pulses are frequently used in MRI for applications such as targeted excitation, fat–water separation or metabolic imaging with hyperpolarised ^{13}C compounds. For the design, the problem is typically separated into the different dimensions. In this work, a method to directly design two-dimensional pulses using the small-tip angle approximation is introduced based on a direct matrix representation. The numerical problem is solved in a single step directly in two dimensions by matrix inversion. Exemplary spectral–spatial excitation and spatio-temporal encoding (SPEN) pulses are designed and validated. The main benefits of the direct design approach include a reduction of artefacts in case of spectral–spatial pulses, a simple and straightforward computer implementation and high flexibility in the pulse design.

1. Introduction

Multi-dimensional radio-frequency (RF) pulses are used in special applications such as those involving targeted or spectral–spatial (SPSP) excitation [1–4]. A recent application of SPSP pulses is metabolic imaging of hyperpolarised 13C compounds, which demands highly efficient sampling of the decaying and non-replenishing magnetisation. A single metabolite is excited spectrally selectively in a single slice, hence not requiring spectral encoding during the acquisition. The image can be encoded by sampling the metabolite's spatial dimensions for example with a single-shot imaging readout such as EPI or spirals [5–8].

Another class of pulses are pulses with a quadratic phase. Overlaying a quadratic phase onto the profile leads to an approximately quadratic phase in the other domain, the pulse coefficients, as well [9]. The introduction of the quadratic phase can be used to spread out the central main lobe, thereby reducing the peak B_1 amplitude of that pulse. The quadratic phase is approximately the same as a linear gradient sweep and can be used for spatio-temporal encoding (SPEN) [10,11]. The SPEN concept works by exciting spins sequentially in time with a linear frequency sweep. Together with a linear gradient, this sweep translates into a spatially sequential excitation, which can be read out directly with a gradient of opposite polarity. The use of SPEN does not require Fourier transformation normally used for image reconstruction. The spatial information is extracted from the magnitude of the acquired data, while the spectral information can be reconstructed from the phase of the acquired data. Two-dimensional (2D) pulses are advantageous for slice-selective imaging [11].

Commonly, 2D pulses are designed using the so-called separable design, by first choosing a suitable gradient trajectory, and subsequently designing a 1D spectral (or quadratic phase) and a 1D spatial filter function and finally combining this into the actual 2D pulse, possibly using a correction function [5]. In this work, we introduce a simple 2D pulse design approach using direct matrix inversion, which reduces sideband artefacts. Furthermore, exemplary SPSP and SPEN pulses are generated to demonstrate the benefits of the design. Three different kinds of "all purpose" SPSP pulses are designed for metabolic imaging with hyperpolarised pyruvate: (1) pulse with bidirectional gradient modulation and its excitation shifted to the first sidelobes; (2) pulse with bidirectional gradient modulation and its excitation centred; and (3) pulse with flyback gradient modulation and its excitation centred. These three pulses are implemented on the MR scanner and the resulting profile and performance are validated using phantoms. These results are compared to simulations obtained by solving the Bloch equations. Two different kind of SPEN pulses are designed: one with a Cartesian gradient modulation and a quadratic phase along the slower, phase-encoded direction; and one with a spiral trajectory and a quadratic phase in both directions.

Abbreviations: FOV, field of view; MRS, magnetic resonance spectroscopy; RF, radio-frequency; ROI, region of interest; SNR, signal-to-noise ratio; SPSP, spectral–spatial; SPEN, spatio-temporal encoding; EPI, echo-planar imaging; CSI, chemical-shift imaging.

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2. Theory and methods

2.1. Spectral–spatial pulses

Spectral–spatial (SPSP) pulses are 2D radio-frequency (RF) pulses that are used for combining frequency- and slice-selective excitation, as shown in Fig. 1. Typically, a large bandwidth in the spatial dimension and a narrow bandwidth in the spectral dimension is required. This naturally leads to using the sublobes for the spatial dimension and the overall pulse envelope for the spectral dimension Fig. 1. Hence, the spatial dimension is quasi-continuously sampled, while sidelobe artefacts will occur along the spectral dimension in the excitation profile due to its coarse sampling [12]. Sidelobe artefacts in the spatial dimension will be outside the region of interest if the discretisation frequency is high enough (e.g., on the used MR scanner 500 MHz).

The SPSP pulse design starts by defining the number n and duration of the pulse's sublobes τ, and by choosing the kind of gradient modulation, bidirectional or flyback, for applying RF during both or only one gradient polarity, respectively. With a maximum gradient strength $g_{\max}$ and slew rate $s_{\max}$ given by the scanner hardware, the gradient trajectory is determined, as the gradients are typically driven at their limits. The minimum slice thickness z determines the spatial bandwidth

$$BW = \frac{\gamma}{2\pi} \cdot z \cdot g_{\max}, \tag{1}$$

which together with the duration of the sublobes determines the spatial time-bandwidth product (measure of the quality of the pulse profile). The overall pulse duration is given by the product of duration and number of sublobes plus the spatial rewinder $T = n\tau + t_{\text{rewinder}}$. The overall duration is the main constraint for the spectral time-bandwidth product.

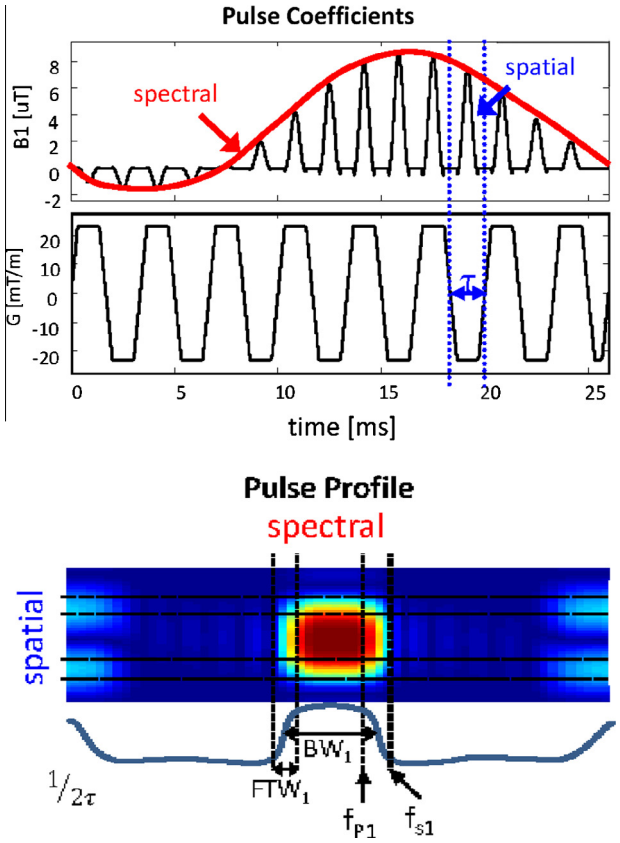

Fig. 1. Principles of spectral–spatial (SPSP) excitation pulses. The pulse coefficients are shown on top, while the corresponding excitation profile is shown below. The overall envelope of the pulse determines the spectral domain, while the individual sublobe determine the spatial dimension. Different metabolites and slices can be acquired in subsequent excitations by modulating the frequency accordingly.

Choosing these parameters involves a delicate trade-off; the duration of sublobes τ should be sufficiently long in order to obtain a suitable spatial time-bandwidth product. At the same time, τ determines the position of the spectral sidelobe artefacts stemming from the coarse sampling. These artefacts should not overlap with other peaks, ideally lying outside the spectral region of interest, hence requiring a short duration of the sublobes τ. The overall pulse duration T should be sufficiently long to obtain a suitable spectral profile, but sufficiently short for reducing artefacts such as sensitivity to flow or decay due to T_2^*. The overall quality of the pulse is determined by the time-bandwidth products; however, error ripples can be traded off against selectivity. A more selective pulse with a narrow transition band will have larger error ripples and vice versa.

The 2D excitation profile b_m needs to be chosen in alignment with the constraints of the overall parameter selection mentioned above. In other words, the profile must be physically realisable, as otherwise the fitting errors will be too large. The desired excitation profile is given by

$$b_m = \begin{cases} 1 & \text{for } |f_{1,m}| < f_{P1} \text{ and } |f_{2,m}| < f_{P2}, \\ 0 & \text{for } |f_{1,m}| > f_{S1} \text{ or } |f_{2,m}| > f_{S2}, \end{cases} \tag{2}$$

with the passband and stopband frequencies being defined by $f_P = \frac{1}{2} BW(1 - FTW)$ and $f_S = \frac{1}{2} BW(1 + FTW)$, respectively. The parameter $BW = F_P + F_s$ denotes the bandwidth and $FTW = \frac{f_S - f_P}{BW}$ denotes the fractional transition width. The transition band between f_P and f_S is undefined and no sampling points are included in this band for the fit. For the "shifted" pulse, this profile is shifted in the spectral dimension to the position of the first sidelobe. The time reference of all designed pulses in the spectral profile is shifted towards the end of the pulse, thereby leading to some self-refocusing in the spectral domain. This leads to the formation of a main lobe towards the end of the pulse, which reduces relaxation, flow and motion effects.

The excitation matrix A describes the forward model of the excitation process in the small tip-angle approximation and is composed of k-space locations and the discrete spatial and spectral frequencies $f_{1,m}$ and $f_{2,m}$, respectively. This matrix is given by

$$A_{m,n} = \exp\left(-2\pi i (f_{1,m} k_{1,n} + f_{2,m} k_{2,n})\right). \tag{3}$$

The spatial k-space locations are obtained by integrating the zig-zag gradient modulation $k_{1,n} = \frac{\gamma}{2\pi} \int_T^t G(t')dt'$. The spectral k-space locations for a pulse of duration T are given by $k_{2,n} = T - t$. The $k$space modulation follows the convention in [12] and is shown in Fig. 2 for both bidirectional and flyback gradient modulation. The sampling points along the spatial and spectral frequencies $f_{1,m}$ and $f_{2,m}$ have to approximately fulfil the Nyquist criterion. Because the k-space is not equidistant and because of the undefined transition region, oversampling is required. The sampling density used in this work is slightly non-uniform, placing more sampling points $f_{1,m}$ and $f_{2,m}$ within the central part of the excitation profile and near the transition region in order to move some of the fitting errors to the sides.

Three general-purpose SPSP pulses were designed for metabolic imaging of [1-13C]pyruvate and its downstream metabolites: (1) a spectrally shifted and (2) centred profile both with bidirectional gradient modulations, and (3) a spectrally centred profile with flyback gradient modulation. The requirements for this metabolic imaging application are that the spectral profile is suitable for exciting a single resonance of any of the five present metabolites (pyruvate, lactate, alanine, bi-carbonate and pyruvate-hydrate)