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Shielded resistive electromagnets of arbitrary surface geometry using the boundary element method and a minimum energy constraint

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ABSTRACT

Eddy currents are generated in MR by the use of rapidly switched electromagnets, resulting in time varying and spatially varying magnetic fields that must be either minimized or corrected. This problem is further complicated when non-cylindrical insert magnets are used for specialized applications. Interruption of the coupling between an insert coil and the MR system is typically accomplished using active magnetic shielding. A new method of actively shielding insert gradient and shim coils of any surface geometry by use of the boundary element method for coil design with a minimum energy constraint is presented. This method was applied to shield *x*- and *z*-gradient coils for two separate cases: a traditional cylindrical primary gradient with cylindrical shield and, to demonstrate its versatility in surface geometry, the same cylindrical primary gradients with a rectangular box-shaped shield. For the cylindrical case this method produced shields that agreed with analytic solutions. For the second case, the rectangular box-shaped shields demonstrated very good shielding characteristics despite having a different geometry than the primary coils.

1. Introduction

Almost all applications in magnetic resonance require rapid switching of magnetic gradient fields within the scanner, and many emerging applications take advantage of rapidly switched magnetic shim coils. Eddy currents are generated in the conducting structures of the MR system by the use of these room temperature, resistive, electromagnets, resulting in time varying and spatially varying magnetic fields that must be either minimized or corrected. This problem is further complicated when non-cylindrical or asymmetrical gradient or shim insert systems are used for specialized applications [1–3]. Asymmetric coils generate more complex eddy currents, which generally produce non-linear magnetic fields within the system. Beyond effects on image quality, current (and therefore power deposition) induced in the cold structures of the MR system cause increased helium boil-off. Rapidly switched electromagnets (particularly even-order zonal shim coils and similar devices) coupling with the superconducting coils of the main magnet may result in decreased system stability and, in the most extreme cases, quenching of the superconducting system.

Minimization of the coupling between an insert coil (gradient or otherwise) and the MR system is typically accomplished using active magnetic shielding [4]. Active shielding is a technique that makes use of a second coil, usually driven in series with the primary coil, to cancel the field effects of the primary coil over a desired region. The standard shielded gradient coil consists of a cylindrical primary coil, coaxial with a shielding coil set at a larger radius such that the net inductive coupling with the rest of the MR system is greatly reduced. The use of non-cylindrical and asymmetric gradients has made the design of active shielding systems both more challenging and yet more important.

Many methods for designing shielding coils have been used in the past. These methods can be divided into two broad categories: analytic methods [5,6], and purely numerical methods [2,7–15].

Analytic methods solve for the continuous current density on the shielding surface required to cancel the field over a chosen region. These solutions commonly require the existence of a separable solution to Greens function for the geometry in question, or that symmetry can be exploited in some way. Because of this, extension of these methods from geometrically simple systems [5,16–22] to more complex surface geometries is extremely difficult.

As they are not limited in surface geometry, numerical methods are much more convenient for the design of novel, shielded, insert coils. A particular numerical design method that has been extremely successful is the boundary element (BE) method [2,10–15]. Advances in this method have allowed the inclusion of engineering constraints, such as minimum wire spacing and gaps for electrical and cooling connections, into the design process [13–15,23].

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However, arriving at optimal shielded coil designs using this technique is challenging. Depending on the sophistication and method of parameterization, computational power, computing time, and even convergence can be problems.

To date, there are three methods for designing shielded coils using the BE method each with their own problems, they are: (1) by specifying additional target points outside of the shield geometry to have zero field; (2) inclusion of an additional "bore" surface and minimizing the field produced (in the region of interest) by currents induced in the bore by switching of the gradient coil [12,24]; and (3) Once again, including an additional bore surface but now minimizing the power deposited into this surface by the induced currents.

The first method can produce different shielded coil designs depending on the distance of the target points to the shield surface. The second and third methods cannot be used for retroactive shield design and introduce an additional design parameter, which results in a drastic increase in computation time and optimization complexity. Furthermore, depending on the field minimization method used, the second method has the additional problem that a large amount of current can be induced on the specified bore surface (corresponding to bad shielding) but the "eddy fields" produced will cancel out at the region of interest, leading to a false sense of shielding ability [24].

In this work, a new approach to designing shielded resistive electromagnets (i.e. gradient and shim coils) using the boundary element method is presented. This method is based upon the assumption that the optimal shield for a given primary current density will minimize the total energy of the primary and shield system. The method is first presented as a means to create retroactive shield designs for known primary wire patterns. Next, the mathematics are described to incorporate the technique into the BE method design and optimization procedure as a shielding constraint, allowing the shield design to be created in tandem with the primary current density. To test performance, the shielding solution of this method is compared against an analytical method for the case of a cylindrical gradient coil. To highlight the versatility of the approach in design over non-traditional surface geometries, a complex (though impractical) case of shielding a cylindrical gradient coil with a rectangular box-shaped coil is shown. The resulting performance of the shielded gradient coils produced using the non-traditional box shield geometry is compared to the completely cylindrical case.

2. Methods

2.1. Boundary element method

In the boundary element method, a surface geometry must be discretized into a finite element mesh. Next, a stream function, and corresponding current density, is defined over the mesh surface. If the stream function is represented as a piecewise linear function over the mesh and we are using triangular mesh elements, the current density over the surface is approximated by:

$$\psi(\mathbf{r}) \approx \sum_{n=1}^{N} I_n \psi_n(\mathbf{r}) \tag{1}$$

$$\mathbf{J}(\mathbf{r}) = \nabla \times [\psi(\mathbf{r})\mathbf{n}(\mathbf{r})] \approx \sum_{n=1}^{N} I_n \nabla \times [\psi_n(\mathbf{r})\mathbf{n}(\mathbf{r})] = \sum_{n=1}^{N} I_n \mathbf{J}_n(\mathbf{r}) \tag{2}$$

where $\psi(\mathbf{r})$ is the stream function, $\mathbf{n}(\mathbf{r})$ is the normal of the mesh surface, I_n is the weighting coefficient for the stream function at node n of the mesh, and $\mathbf{J}_n(\mathbf{r})$ is the current density basis function for node n. The basis functions are described in [11] and are composed of a sub-set of vectors, $\mathbf{v}_{nj}$, associated with each triangular element ($j = 1 \rightarrow N_n$) containing node n as a vertex. $\mathbf{v}_{nj}$ is equal to the edge vector opposite node n divided by twice the elemental area.

All coil properties that can be found using the current density (e.g. magnetic field, power, torque) can now be described by the current basis functions along with their weighting coefficients. In order to design an electromagnet with this method, one must create and minimize a functional. The functional can contain a uniformity term, a power term, a magnetic energy term, a shielding term, torque constraints, etc. A very simple minimum power functional is:

$$U = \frac{1}{2}\sum_{k=1}^{K}\left[B_z(\mathbf{r}_k) - B_z^{tar}(\mathbf{r}_k)\right]^2 + \frac{\beta}{2}P \tag{3}$$

$$U = \frac{1}{2}\sum_{k=1}^{K}\left[I_n c_n(\mathbf{r}_k) - B_z^{tar}(\mathbf{r}_k)\right]^2 + \frac{\beta}{2}I_n I_m R_{nm} \tag{4}$$

where $B_z(\mathbf{r}) = I_n c_n(\mathbf{r})$ is the z-component of the magnetic field produced by the coil (the matrix $c_n(\mathbf{r})$ is described in [11]), $B_z^{tar}(\mathbf{r})$ are the user specified magnetic field targets, P is the power deposition in the coil, R_{nm} is the resistance matrix of the mesh surface, and β is the user specified weighting coefficient between coil power and field uniformity. Minimizing this functional with respect to I_n and solving gives:

$$I_n = \left[c_n(\mathbf{r})c_m(\mathbf{r}) + \beta R_{nm}\right]^{-1}\left[c_m(\mathbf{r})B_z^{tar}(\mathbf{r})\right] \tag{5}$$

Which are the stream function weighting coefficients over the surface. Design optimization is achieved by appropriately adjusting the relative weighting values between performance parameters (e.g. power, energy, torque, uniformity, shielding performance).

2.2. Retroactive shield design

The total current density for the primary and shield system can be decomposed into its individual components:

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_{\mathrm{p}}(\mathbf{r}) + \mathbf{J}_{\mathrm{s}}(\mathbf{r}) \tag{6}$$

where $\mathbf{J}_{\mathrm{p}}(\mathbf{r})$ and $\mathbf{J}_{\mathrm{s}}(\mathbf{r})$ represent the primary and shielding current densities respectively. Using the approximation of (2), the current densities can be expressed over their respective meshes as:

$$\mathbf{J}_{\mathrm{p}}(\mathbf{r}) \approx \sum_{n=1}^{N} I_{\mathrm{p}n}\mathbf{J}_{\mathrm{p}n}(\mathbf{r}) \tag{7}$$

$$\mathbf{J}_{\mathrm{s}}(\mathbf{r}) \approx \sum_{m=1}^{M} I_{\mathrm{s}m}\mathbf{J}_{\mathrm{s}m}(\mathbf{r}) \tag{8}$$

With this current density approximation, the total magnetic energy of the primary and shield system is:

$$E = \frac{1}{2}I_{\mathrm{p}n}I_{\mathrm{p}m}L_{\mathrm{p}nm} + \frac{1}{2}I_{sq}I_{sk}L_{sqk} + I_{\mathrm{p}n}I_{sk}M_{\mathrm{ps}nk} \tag{9}$$

where L_{nm} is the self inductance matrix of the surface, described in [11] (with subscripts "p" and "s" denoting primary and shield respectively), and $M_{\mathrm{ps}nk}$ is the mutual inductance matrix between the primary and shield surfaces calculated by the expression:

$$M_{\mathrm{ps}nk} = \frac{\mu_0}{4\pi}\int_S \int_{S'} \frac{\mathbf{J}_{\mathrm{p}n}(\mathbf{r}) \cdot \mathbf{J}_{sk}(\mathbf{r'})}{|\mathbf{r}_{\mathrm{p}n} - \mathbf{r}'_{sk}|}\mathrm{d}S'\mathrm{d}S \tag{10}$$

Now, in this instance one is interested in designing a retroactive shield (i.e. the primary coil has already been created and the I_{p} values are known). Therefore, the only free variables in Eq. (9) are the shield stream function values I_{s}. Minimizing Eq. (9) with respect to the shield stream function values and solving gives: