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Electromagnetic characterization of an MR volume coil with multilayered cylindrical load using a 2-D analytical approach

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ABSTRACT

We present an analytical method for the analysis of Radio Frequency (RF) volume coils for Magnetic Resonance Imaging (MRI), using a 2-D full wave solution with loading by multilayered cylinders. This allows the characterization of radio-frequency E, H, B_1, B_1^+ fields. Comparisons are provided with experimental data obtained at 7.0 T. The procedure permits us to clearly separate the solution to single line source problem (which we call the *primordial solution*) and the composite solution (i.e. *full coil*, i.e. the summations of primordial solutions according to the resonator drive configuration). The capability of separating the primordial solution and the composite one is fundamental for a thorough analysis of the phenomena of dielectric resonance, and of standing wave and multi-source interference. We show that dielectric resonance can be identified only by looking at the electromagnetic field from a single line source.

1. Introduction

Since the emergence of Magnetic Resonance Imaging (MRI) there has been a great interest in predicting and characterizing the electromagnetic behavior of both Radio Frequency (RF) coil and sample under investigations. Birdcage resonators [1,2] and shielded birdcage resonators [3] have become the standard for volume coils. The complex interactions between volume coils and samples cannot be usually solved using analytical methods: thus, numerical methods are adopted. However, customized analytical methods can be developed to calculate these interactions using simplified geometries and assumptions [4–7]. For example, in [4] an approach based on the static solution for homogeneous cylinders irradiated by line current and homogeneous sphere irradiated by a loop is described, while the method in [5] is based on the solution for a current sheet and is suited for homogeneous cylinders.

Here we propose an analytical approach based on the theory of cylindrical waves irradiated by a filament of radiofrequency current (dynamic solution). We provide the solution of the two-dimensional boundary value problem for a shielded birdcage resonator. The load will be modeled as a cylinder and the complexity will be increased considering multiple (possibly lossy) layers. The procedure permits us to evaluate both E and H fields; it follows that it is suited for investigating specific absorption rate (SAR) and B_1 inhomogeneities in realistic scenarios. Each layer can represent different human tissues, characterized by the equivalent dielectric constant [8]. The multi-layered representation permits us to take into account of many propagation phenomena, including diffraction around a curved lossy surface, reflections off the body, multi-reflection between different tissues and penetration. All of these effects play a role in the electromagnetic behavior, though the relative importance of each effect will depend on factors such as the frequency, polarization, radius of curvature, and tissue properties.

It is worth pointing out that the analysis is not limited to birdcage resonator, but can be equally applied to TEM resonator [3] and in general to virtually any ladder resonator [9]. Since the method resorts to a 2-D full wave solution, end-rings effect cannot be accounted for. In the proposed procedure, the rods are approximated by line sources: obviously, this approximation holds in the region external to the rods only. It should be pointed out that we used a line sources approach for the electromagnetic fields evaluation and not for circuit modeling. The procedure can be applied at any RF frequency since it does not resort to low-frequency or quasi static approximation; thus, it can be used from low to ultra high field MR.

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Comparisons with experimental data obtained at 7.0 T are provided to validate the procedure. The approach is computationally quick (it usually takes a few seconds on moderate performance laptop). Moreover, it permits us to have a complete characterization of RF fields E, H, B_1, B_1^+ under any drive conditions. The procedure allows us to clearly separate the solution to single line source problem (that we will call *primordial solution*) and the composite one (*full coil*, i.e. the summations of primordial solutions according to the resonator drive configuration). This feature is important, particularly since the primordial solution yields insights into the physical phenomena of dielectric resonance and standing wave effects.

2. Theory

2.1. Eccentrically placed line current

Consider an electric current line source I positioned, using cylindrical coordinates, at (ρ', ϕ'). The electric current line is parallel to the z-axis and radiates into free space. We obtain the E and H fields expressed in time-harmonic phasor notation by setting the vector potential to $\overrightarrow{A} = \psi \hat{i}_z$ and using the following relations [10] in cylindrical coordinates (see Appendix A for further details):

$$\overrightarrow{E}(\overrightarrow{\rho}) = \begin{bmatrix} \frac{1}{j\omega\varepsilon_0}\frac{\partial^2\psi}{\partial\rho\partial z}\hat{i}_\rho \\ \frac{1}{j\omega\varepsilon_0}\frac{1}{\rho}\frac{\partial^2\psi}{\partial\phi\partial z}\hat{i}_\phi \\ \frac{1}{j\omega\varepsilon_0}(\frac{\partial^2}{\partial z^2}+k_0^2)\psi\hat{i}_z \end{bmatrix}; \quad \overrightarrow{H}(\overrightarrow{\rho}) = \begin{bmatrix} \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\hat{i}_\rho \\ -\frac{\partial\psi}{\partial\rho}\hat{i}_\phi \\ 0\hat{i}_z \end{bmatrix} \tag{1}$$

with ε_0 being the free-space electric permittivity, $\omega = 2\pi f$ with f the operation frequency and k_0 the free-space wave number. Then:

$$\overrightarrow{A} = \frac{I}{4j}H_0^{(2)}(k_0|\overrightarrow{\rho} - \overrightarrow{\rho}'|)\hat{i}_z \tag{2}$$

where $H_0^{(2)}$ is the Hankel function of second kind and zero order. By means of the addition theorem [11], we have

$$H_0^{(2)}(k_0|\overrightarrow{\rho} - \overrightarrow{\rho}'|) = \begin{cases} \sum_{n=-\infty}^{n=+\infty} H_n^{(2)}(k_0\rho')J_n(k_0\rho)e^{jn(\phi-\phi')} & \text{for } \rho < \rho' \\ \sum_{n=-\infty}^{n=+\infty} H_n^{(2)}(k_0\rho)J_n(k_0\rho')e^{jn(\phi-\phi')} & \text{for } \rho > \rho' \end{cases} \tag{3}$$

where $H_n^{(2)}$ represents the Hankel function of second kind and nth-order, and J_n is the Bessel function of nth-order.

2.2. Homogeneus cylindrical load

Consider now a homogeneous dielectric cylinder positioned at the center of the reference system, having radius a_0, irradiated by the line source ($a_0 < \rho'$), as in Fig. 1a. The cylinder is characterized by relative constants ε_{r1}, μ_{r1}, and by conductivity σ_1. It holds (with $\mu_{r1} = 1$):

$$\begin{cases} \varepsilon_1 = \varepsilon_0\varepsilon_{eq1} \\ \mu_1 = \mu_0 \end{cases}, \quad \text{with} \quad \varepsilon_{eq1} = \left(\varepsilon_{r1} - j\frac{\sigma_1}{2\pi f\varepsilon_0}\right) \tag{4}$$

From Eq. (4) it is evident that, if $\sigma_1 \neq 0$, ε_{eq1} varies with frequency. It is worth pointing out that, in human tissues, the frequency-dependence of ε_{r1} and σ_1 has to be taken into account as well. Next, we define the complex wave number inside the cylinder:

$$k_1 = \sqrt{(2\pi f)^2\varepsilon_1\mu_1} \tag{5}$$

We proceed by writing the following two equations that describe the E field in free-space and inside the cylinder, respectively (see the Appendix A for details):

$$\begin{aligned} E_z^0 &\propto \sum_{n=-\infty}^{n=+\infty} H_n^{(2)}(k_0\rho')J_n(k_0\rho)e^{jn(\phi-\phi')} + \sum_{n=-\infty}^{n=+\infty} a_n H_n^{(2)}(k_0\rho)e^{jn(\phi-\phi')} \\ E_z^1 &\propto \sum_{n=-\infty}^{n=+\infty} b_n J_n(k_1\rho)e^{jn\phi} \end{aligned} \tag{6}$$

while the other components of $\overrightarrow{E}$ are zero, as it follows from Eq. (1). Similar equations hold for the H field:

$$\begin{aligned} H_\rho^0 &\propto \frac{e^{jn\phi}}{\rho}\sum_{n=-\infty}^{n=+\infty} H_n^{(2)}(k_0\rho')J_n(k_o\rho)e^{jn(\phi-\phi')} + \frac{e^{jn\phi}}{\rho}\sum_{n=-\infty}^{n=+\infty} a_n H_n^{(2)}(k_0\rho)e^{jn(\phi-\phi')} \\ H_\rho^1 &\propto \frac{e^{jn\phi}}{\rho}\sum_{n=-\infty}^{n=+\infty} b_n J_n(k_1\rho)e^{jn\phi} \end{aligned} \tag{7}$$

and

$$\begin{aligned} H_\phi^0 &\propto -\sum_{n=-\infty}^{n=+\infty} H_n^{(2)}(k_0\rho')J_n'(k_o\rho)e^{jn(\phi-\phi')} - \sum_{n=-\infty}^{n=+\infty} a_n H_n'^{(2)}(k_0\rho)e^{jn(\phi-\phi')} \\ H_\phi^1 &\propto -\sum_{n=-\infty}^{n=+\infty} b_n J_n'(k_1\rho)e^{jn\phi} \end{aligned} \tag{8}$$

while the other component of $\overrightarrow{H}$ is zero, as it follows from Eq. (1). Thus, $\overrightarrow{H}$ lies entirely in the transverse plane.

All coefficients can be determined by imposing the boundary conditions of $\overrightarrow{E}$ and $\overrightarrow{H}$ on the surface of the cylinder and by using the orthogonality of the exponential functions: thus, $\overrightarrow{E}$ and $\overrightarrow{H}$ can be determined in each region. The previous formulation has been derived assuming the dielectric cylinder positioned in the center of the reference system. However, the procedure can be extended to eccentric cylinders arbitrarily displaced with respect to the reference system [12]. Eqs (6)–(8) are represented through a summation with n spanning from $-\infty$ to $+\infty$. For computation, the summation is truncated after $2N + 1$ terms, i.e. with n going from

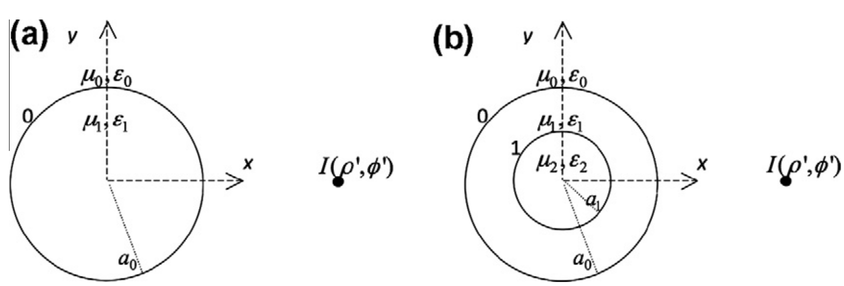

Fig. 1. Homogeneous cylinder (a) and two-layer stratified concentric dielectric cylinder (b) in free space irradiated by a line source. The homogeneous cylinder is positioned at the center of the reference system, it has radius a_0 and the surface is denoted with 0. The two-layer stratified concentric cylinder is positioned at the center of the reference system; the outer layer has radius a_0; the inner layer has radius a_1; the outer and inner surfaces are denoted with 0 and 1, respectively.