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Communication

A Bayesian approach to characterising multi-phase flows using magnetic resonance: Application to bubble flows

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ABSTRACT

Magnetic Resonance (MR) imaging is difficult to apply to multi-phase flows due to both the inherently short T_2^* characterising such systems and the relatively long time taken to acquire the data. We develop a Bayesian MR approach for analysing data in *k*-space that eliminates the need for image acquisition, thereby significantly extending the range of systems that can be studied. We demonstrate the technique by measuring bubble size distributions in gas–liquid flows. The MR approach is compared with an optical technique at a low gas fraction (~2%), before being applied to a system where the gas fraction is too high for optical measurements (~15%).

1. Introduction

Magnetic resonance (MR) is being increasingly applied to study multi-phase flows due to its ability to study optically opaque systems non-invasively. However, MR is an inherently slow technique with conventional imaging approaches requiring several minutes to acquire an image. A variety of fast imaging techniques are available [1–3], however these are still not quick or robust enough to study multi-phase flows which are characterised by rapid temporal variations, high shear rates and short relaxation times. This paper presents a new method of characterising multi-phase flows by re-posing the MR experiment as a Bayesian analysis problem that does not require image acquisition. Such an approach is advantageous for dynamic systems and could also be applied to low sensitivity portable MR systems. The procedure is developed and demonstrated for sizing gas bubbles in liquid flows.

Accurate measurement of bubble size is critical to improving our understanding of fundamental physical phenomena in multi-phase flows including turbulent drag, bubble coalescence, and heat transfer. However, the sizing of bubbles, particularly in high volumetric gas fraction flows, remains challenging. Measurements are currently made using optical, electrical, and light scattering techniques, amongst others [4–6]; these techniques have their limitations. Invasive techniques distort the local bubble size and shape. Non-invasive techniques are limited to low gas fraction ($\lesssim$5%) systems or near wall observations because of the increased light scattering by dense bubble swarms and interference effects between neighbouring bubbles. The Bayesian approach developed in this work is applicable to high gas fraction measurements (up to ~50%), and thus enables measurements of systems that were previously impossible to study.

Bayesian analysis has previously been used in a variety of MR applications [7–10]. It has been shown to improve the recovery of an MR spectrum from noisy data [7] and to improve the accuracy of flow measurements by enabling a sparse sampling procedure to be used [9]. In this work we exploit both these advantages of Bayesian analysis to enable measurements of the bubble size distribution in a dynamic system.

The approach used is derived from texture analysis concepts in image processing [11] and extends previous approaches for analysing MR data [12] to provide quantitative measurements in dynamic systems. The signal measured using MR, $S(k)$, is governed by:

$$S(k) = \int \rho(x)\exp(i2\pi kx)dx, \tag{1}$$

where $\rho(x)$ defines the image (e.g. liquid map), x corresponds to the spatial position and $k = \gamma(2\pi)^{-1}\int G_x(t)dt$, where γ is the gyromagnetic ratio and $G_x(t)$ describes the strength of the magnetic field gradient in the x-direction as a function of time, t. Thus, the signal, $S(k)$, and image, $\rho(x)$, are mutually conjugate Fourier pairs; hence by controlling the gradient strength as a function of time it is possible to sample any point in the spatial frequency domain of the image, commonly referred to in MR literature as *k*-space [13].

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In conventional MR, an image of the system is obtained by measuring a signal in k-space and then taking a discrete Fourier transform of these data. This image would subsequently be analysed to obtain the desired information, for example a bubble size distribution. However, multi-phase flows will often change over a time scale less than that required to acquire an image, leading to misregistration artefacts in the image that make the subsequent analysis challenging, inaccurate and frequently impossible.

In the Bayesian approach proposed in this paper, a likelihood function is developed relating the measured signal, $S(k)$, to the state of the system θ (e.g. the bubble size distribution). This eliminates the conventional imaging requirement that the system is stable for the duration of the acquisition, requiring only that θ is constant during the experiment. In this specific case of a bubble size distribution, the size and location of individual bubbles will change over a time scale of the order of a few milliseconds. However, the overall distribution of bubble sizes will remain stable over time, as it is determined by only the fluid properties, system design and operating conditions [14]. Therefore, a system that cannot be studied by image acquisition can be studied using a Bayesian methodology. The Bayesian approach reported here enables the characterisation of the size distribution of approximately spherical objects in multiphase systems. Therefore, in addition to the example of bubble sizing reported here, this same analysis could be applied to, for example, emulsion droplet sizing, droplet sizing of sprays, and the determination of pore size in porous media. The present case study was selected because the measurement of bubble size distributions in gas–liquid flows above a gas fraction of 5% cannot be made by optical techniques and represents a significant measurement challenge. We develop the technique and present results from numerical simulations of bubble size distributions with both ideal and noisy data. Numerical simulations suggest that the technique is applicable up to a gas fraction of 50%. We then compare experimental measurements of the bubble size distribution at low gas fraction (~2%) with an optical technique, before presenting measurements of the bubble size distribution at a gas fraction of ~15%, which is in excess of that which can be measured optically.

2. Model development

In Bayesian analysis the state of a system θ is inferred from a set of observations $\hat{y}$ from the posterior probability density function $p(\theta|\hat{y})$:

$$p(\theta|\hat{y}) \propto p(\hat{y}|\theta)p(\theta), \tag{2}$$

where $p(\hat{y}|\theta)$ is the likelihood function and $p(\theta)$ incorporates prior knowledge. In this work we are attempting to determine the size distribution of bubbles, which corresponds to θ, given a set of measurements, $\hat{y}$, of the signal intensity in k-space. In the approach described here, we assume a functional form for the size distribution and estimate the parameters of that distribution. Thus, if the radius of an individual bubble is r, then we characterise the distribution of r by modelling it using two parameters, the mean radius $\bar{r}$ and a standard deviation σ_r. These two parameters describe the state of the system θ, which we obtain as $p(\theta|\hat{y})$. We present results for two cases: (i) bubbles of a single size, i.e. $\sigma_r = 0$ and $r_j = \bar{r}$ for all bubbles j and (ii) a bubble size distribution given by a log-normal distribution, where the log-normal distribution is defined by:

$$p(r, \mu, \sigma_\mu) = \frac{1}{r\sigma_\mu\sqrt{2\pi}} \exp\left(-\frac{(\ln r - \mu)^2}{2\sigma_\mu^2}\right), \tag{3}$$

and the parameters μ and σ_μ uniquely define the mean $\bar{r} = \exp\left(\mu + \sigma_\mu^2/2\right)$ and variance $\sigma_r^2 = \bar{r}^2\left(\exp\left(\sigma_\mu^2\right) - 1\right)$ of the distribution. We use a log-normal distribution as this is observed empirically [4,5]. In each case, the calculated posterior distribution characterises the probability distribution for the parameters $\bar{r}$ and σ_r.

The likelihood function is determined by considering how the signal intensity varies in k-space given a particular distribution of bubble sizes and bubble shape. We begin by formulating a 1D image $f(x)$ which comprises the projection of N bubbles onto the x-axis. The projection of each individual bubble is defined by a function $h(r,x)$, where r is the characteristic size of the bubble and x is the spatial coordinate. Then, defining the Fourier transforms of $f(x)$ and $h(r,x)$ as $F(k)$ and $H(r,k)$, respectively, the signal measured by MR obeys:

$$F(k) = \sum_{j=1}^{N} H(r_j, k) \exp(-i2\pi k x_{c,j}), \tag{4}$$

where $x_{c,j}$ is the location of the centre of the jth bubble and use has been made of the linearity and shift invariance of the Fourier transform. As an example, Fig. 1 shows (a) $f(x)$ for a simulation of 30 identical spherical bubbles and (b) the corresponding magnitude of $F(k)$, the discrete Fourier transform of these data. Assuming $\{x_{c,j}\}$ is independent and identically distributed, then for a given k, the expected $F(k)$ is 0, i.e. $E(F(k)) = 0$ and

$$E(|F(k)|^2) = NE(|H(r_j, k)|^2). \tag{5}$$

If a signal is obtained from the magnitude of a sum of complex values, each of random phase, this signal will be described by the Rayleigh distribution [15], provided the number of values in the sum is sufficiently large. Therefore, in the limit of large numbers (i.e. large N), the likelihood function describing the magnitude of the signal at any given k will be defined by a Rayleigh distribution:

$$p(|F(k)||\lambda) = \frac{|F(k)|}{\lambda(k)^2} \exp\left(-\frac{|F(k)|^2}{2\lambda(k)^2}\right), \tag{6}$$

where $\lambda^2 = E(|F(k)|^2)/2$. The value of N required for Eq. (6) to hold will depend on the distribution of bubble sizes in the system. For bubbles of a uniform size, $E(|H(r_j, k)|^2) = |H(\bar{r}, k)|^2$ and therefore Eq. (6) holds for $N \geqslant 2$. For a bubble size distribution given by a log-normal distribution the value of N required will depend on the parameters of the distribution and an expression similar to Eq. (6) can be derived numerically; for log-normal distributions with $\sigma_r < 0.5\bar{r}$, numerical results show that Eq. (6) holds for realistic experimental values of N (i.e. $N \geqslant 6$).

Eq. (6) defines the likelihood function for the signal as a function of $\bar{r}$ and σ_r. Therefore, if λ^2 is known for all $\bar{r}$ and σ_r, $p(\theta|\hat{y})$ is obtained from Eq. (2), and this can be used to estimate $\theta \equiv \{\bar{r}, \sigma_r\}$ which

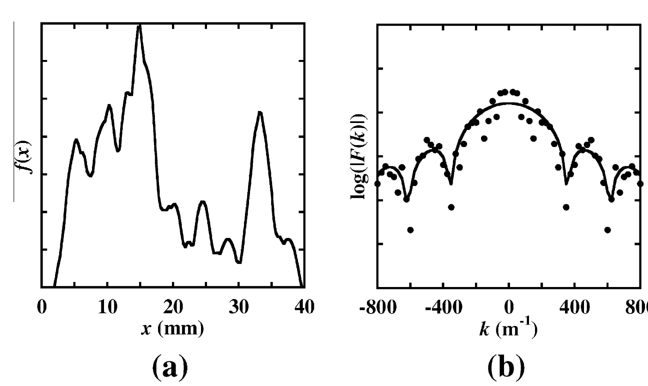

Fig. 1. (a) Plot of the projection onto the x-axis of 30 spheres each of radius 2 mm and (b) (•) the magnitude of $|F(k)|$ and (–) $E(|F(k)|)$ of the discrete Fourier transform of the data shown in (a). The intensity on the vertical axis in (a) is proportional to $\int\int \rho(x, y, z)dydz$. The solid line in (b) is derived from Eqs. (6) and (7).