Heteronuclear decoupling by optimal tracking

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ABSTRACT

The problem to design efficient heteronuclear decoupling sequences is studied using optimal control methods. A generalized version of the gradient ascent engineering (GRAPE) algorithm is presented that makes it possible to design complex non-periodic decoupling sequences which are characterized by tens of thousands of pulse sequence parameters. In contrast to conventional approaches based on average Hamiltonian theory, the concept of optimal tracking is used: a pulse sequence is designed that steers the evolution of an ensemble of spin systems such that at a series of time points, a specified trajectory of the density operator is tracked as closely as possible. The approach is demonstrated for the case of low-power heteronuclear decoupling in the liquid state for *in vivo* applications. Compared to conventional sequences, significant gains in decoupling efficiency and robustness with respect to offset and inhomogeneity of the radio-frequency field were found in simulations and experiments.

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1. Introduction

Heteronuclear decoupling methods have a long history in NMR spectroscopy [1–18]. The goal of broadband heteronuclear decoupling sequences is to collapse a spin *I* multiplet splitting by irradiating a spin *S* that is coupled to *I* in order to simplify the spectra and to increase the signal-to-noise ratio. At the same time, the decoupling sequence should introduce only a minimal amount of artifacts, such as decoupling sidebands. Furthermore, in order to avoid undesirable sample heating or damage to the probe, the radio frequency (rf) power of the decoupling sequence should be as small as possible. This is of particular importance in medical imaging or *in vivo* spectroscopy of humans. The earliest heteronuclear decoupling methods were based on cw irradiation [3] and noise decoupling [4]. Significantly improved decoupling sequences were found based on composite [1,2,5,7–9] or shaped [13–18] inversion pulses in combination with highly compensated cycles and supercycles [1,2,7,20–22]. Theoretical approaches that have been used for the analysis and design of decoupling sequences include average Hamiltonian [6,23] and Floquet [18] theory. Here we introduce a novel approach to decoupling where the decoupling pulse sequence is obtained by optimally tracking the evolution of a density matrix under a desired Hamiltonian. In particular we use this method to optimally track the evolution of a decoupled

Hamiltonian. We show that the optimal control methods introduced in our previous work for transferring the state of the system closest to a desired target state can be generalized to make the evolution of the density matrix track a certain trajectory over an extended time period.

Recently, principles of optimal control theory [24] have found numerous applications in magnetic resonance. Theoretical limits for maximum heteronuclear transfer efficiency have been established for the typical NMR setting, where rf amplitudes can be much larger than heteronuclear couplings. Furthermore, in this limit of strong pulses, time-optimal [25,26] and relaxation-optimized [27,28] pulse sequences were derived, which achieve the theoretical limits. In addition to analytical bounds for the strong pulse limit, optimal-control based algorithms make it possible to numerically explore the physical limits of polarization transfer efficiency in realistic settings, where experimental limitations have to be taken into account. The GRAPE algorithm [29,30] has been successfully applied to problems in liquid-state NMR [31–38], solid-state NMR [39], and quantum information processing [40,41]. In the context of broadband heteronuclear decoupling, robust inversion pulses with minimal rf power [38] are potential candidates for inversion elements suitable for cyclic decoupling sequences. Alternatively, optimal control methods could be used to optimize decoupling elements that produce a desired effective Hamiltonian or an effective propagator over a small time period as demonstrated in [29,39,40]. However, both approaches do not use the full potential of optimal control methods.

Here, we introduce a more general method to design non-periodic, robust low-power decoupling sequences using tracking methods [24,42]. These methods fall into two categories. The first

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one attempts to exactly follow a desired output by some inversion method [43–47]. In the second approach, one tries to find an input control field that will approximate a desired output trajectory. These approximate methods can be constructive [48,49] or they can be based on the solution to an optimal control problem [50–54]. The application of the latter approach to heteronuclear decoupling sequences is presented in the following. The complexity resulting from considering non-periodic decoupling sequences can be handled with reasonable computational cost using efficient numerical methods. Potential applications range from NMR spectroscopy to quantum information processing. Here, we focus on the specific example of low-power decoupling sequences for *in vivo* applications, in order to illustrate the improved performance that can be achieved based on the presented optimal tracking approach.

2. Theory

We consider a system consisting of two heteronuclear spins 1/2 labeled I and S. We assume that spin I is observed while spin S is irradiated by a decoupling sequence. In a doubly rotating frame, the Hamiltonian has the form

$$\mathscr{H}(t) = \mathscr{H}_{off}^I + \mathscr{H}_{off}^S + \mathscr{H}_J^{IS} + \mathscr{H}_{rf}^S(t), \tag{1}$$

with the offset term of spin I

$$\mathscr{H}_{off}^I = 2\pi\nu_I I_z, \tag{2}$$

the offset term of spin S

$$\mathscr{H}_{off}^S = 2\pi\nu_S S_z, \tag{3}$$

the heteronuclear J coupling term

$$\mathscr{H}_J^{IS} = 2\pi J S_z I_z, \tag{4}$$

and the rf term representing the decoupling sequence

$$\mathscr{H}_{rf}^S(t) = 2\pi\epsilon\{u_x(t)S_x + u_y(t)S_y\}. \tag{5}$$

The controls $u_x(t)$ and $u_y(t)$ are the nominal amplitudes of the x and y components of the rf field in the doubly rotating frame and ϵ is a rf scaling factor, which takes into account rf inhomogeneity and miscalibration effects. As $\mathscr{H}_{off}^I$ commutes with the terms $\mathscr{H}_{off}^S$, $\mathscr{H}_J^{IS}$, and $\mathscr{H}_{rf}^S(t)$, the offset effect of spin I can be separated and it is sufficient to consider the simplified Hamiltonian

$$\mathscr{H}'(t) = \mathscr{H}_{off}^S + \mathscr{H}_J^{IS} + \mathscr{H}_{rf}^S(t), \tag{6}$$

for the analysis and design of heteronuclear decoupling sequences, i.e. spin I can be assumed to be on resonance. We assume the initial density operator to be

$$\rho(t_0) = I_x. \tag{7}$$

In the absence of a decoupling sequence, $\mathscr{H}_{rf}^S(t) = 0$, the offset term $\mathscr{H}_{off}^S$ commutes both with $\rho(t_0)$ and with $\mathscr{H}_J^{IS}$ and hence the evolution of the density operator is simply governed by the coupling term $\mathscr{H}_J^{IS}$:

$$\rho(t) = I_x \cos(\pi J t) + 2I_y S_z \sin(\pi J t), \tag{8}$$

and the cosine modulation of the detectable operator I_x results in a doublet with splitting J in the resulting spectrum after Fourier transformation of the time-domain signal corresponding to the expectation value $\langle I_x\rangle(t)$. In typical experimental settings, the signal is not detected continuously, but only at a number of discrete time points T_k. As illustrated in Fig. 1, the digitization of the decoupling sequence (with time slices $\Delta t = t_{m+1} - t_m$) is typically finer than the digitization of the detected signal (with time intervals $\Delta T = T_{k+1} - T_k$) and $M = \Delta T/\Delta t$ is the number of time slices Δt
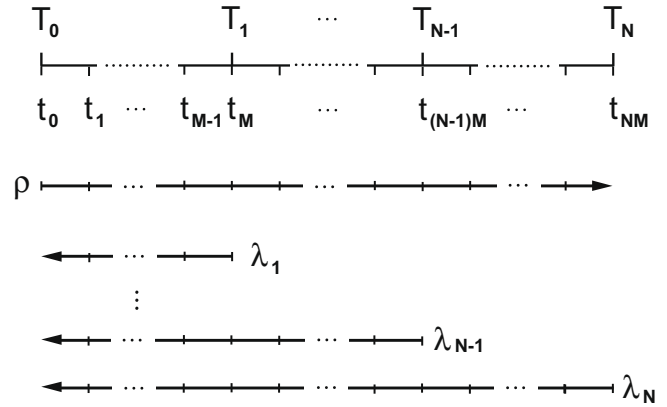
Fig. 1. Schematic representation of the acquisition points T_k (with $0 \leqslant k \leqslant N$) and the digitization of a heteronuclear decoupling sequence, consisting of M time slices $\Delta t = t_{m+1} - t_m$ between two subsequent acquisition points. Rf amplitudes are optimized for a total of N times M time slices. The density operator $\rho(0) = I_x$ evolves forward in time and for each acquisition point, there is a costates $\lambda_k(T_k) = I_x$ that is evolved backward in time. As shown in Appendix 3, the backward evolution of the N costates λ_k can be reduced to a single backward evolution of the combined costate Λ.

per interval ΔT. Hence, if the signal is detected at the $N + 1$ time points $T_0, \ldots, T_N$, the entire decoupling sequence consists of NM time slices and is characterized by NM control amplitudes $u_x(j)$ and by NM control amplitudes $u_y(j)$ with $1 \leqslant j \leqslant NM$. In the case of perfect decoupling, the desired evolution of the density operator is

$$\rho_d(T_k) = I_x \quad \text{for } 0 \leqslant k \leqslant N, \tag{9}$$

i.e. the expectation value $\langle I_x\rangle(t)$ of the detection operator is constant at all time points T_k where a signal is detected. In this case, no splitting is observed in the resulting spectrum after Fourier transformation. Here, we demonstrate how to design heteronuclear decoupling sequences by tracking this desired evolution of the density operator at specified time points T_k, where the signal is detected during the free induction decay. The optimal tracking problem can be efficiently solved using an extended version of the GRAPE algorithm [29]. In the original version of GRAPE, the quality factor for state to state transfer is defined as the projection of the final density operator $\rho(T)$ onto a desired target operator C. The gradient of this quality factor with respect to the control amplitudes (i.e. the pulse sequence parameters) can be calculated efficiently from the density operator $\rho(t)$ that evolves forward in time starting from $\rho(0)$ and a so-called costate operator $\lambda(t)$ that evolves backward in time starting from $\lambda(T) = C$. In the case of heteronuclear decoupling, the projection of the density operator on a series of desired target operators $C(T_k) = I_x$ is relevant for all time points T_k at which the signal is detected. As shown in the following, in this case the gradient of the corresponding quality factor for decoupling can be calculated efficiently from the density operator $\rho(t)$ and a series of costate operators $\lambda_k(t)$ that evolve backward in time starting from $\lambda_k(T_k) = C(T_k) = I_x$ (c.f. Fig. 1).

The quality of decoupling can be quantified by the signal amplitude of the decoupled resonance line in the frequency domain. In addition, the amplitude of decoupling sidebands should be as small as possible. As according to the simplified Hamiltonian $\mathscr{H}'(t)$ (c.f. Eq. (6)) spin I can be assumed to be on-resonance, the amplitude of the spin I spectrum at zero frequency is simply given by the Fourier transform of the free induction decay at frequency $\nu_I = 0$. The detected discrete time-domain signal is

$$s_k = \langle I_x\rangle(T_k) = \text{Tr}\{I_x\rho(T_k)\}, \tag{10}$$