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Communication

Probabilistic Identification and Estimation of Noise (PIESNO): A self-consistent approach and its applications in MRI

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ABSTRACT

Data analysis in MRI usually entails a series of processing procedures. One of these procedures is noise assessment, which in the context of this work, includes both the identification of noise-only pixels and the estimation of noise variance (standard deviation). Although noise assessment is critical to many MRI processing techniques, the identification of noise-only pixels has received less attention than has the estimation of noise variance. The main objectives of this paper are, therefore, to demonstrate (a) that the identification of noise-only pixels has an important role to play in the analysis of MRI data, (b) that the identification of noise-only pixels and the estimation of noise variance can be combined into a coherent framework, and (c) that this framework can be made self-consistent. To this end, we propose a novel iterative approach to simultaneously identify noise-only pixels and estimate the noise standard deviation from these identified pixels in a commonly used data structure in MRI. Experimental and simulated data were used to investigate the feasibility, the accuracy and the stability of the proposed technique.

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1. Introduction

Magnetic resonance imaging (MRI) [1] is a rapidly expanding field and a widely used medical imaging modality possessing many noninvasive and quantitative techniques capable of probing functional activity [2] as well as tissue morphology in the brain [3,4]. Data analysis in MRI is sophisticated and can be thought of as a "pipeline" of closely connected processing and modeling steps.

Because noise in MRI data affects all subsequent steps in this pipeline, e.g., from noise reduction [5] and image registration [6] to techniques for breaking the noise floor [7], parametric tensor estimation [8–11] and error propagation [12–17], accurate noise assessment has an important role in MRI studies.

Noise assessment in MRI usually means the estimation of Gaussian noise variance (or standard deviation (SD)) alone [18–23]. Previously proposed methods for the estimation of Gaussian noise SD can be separated into two groups. In the first group, the Gaussian noise SD is estimated from a manually selected region-of-interest (ROI), while in the second group, it is estimated from an entire image or a volumetric data set automatically without human intervention.

A major problem faced by the first group of *manual* methods is lack of reproducibility of the results. The second group of *automatic* methods overcame this problem by bringing objectivity into the estimation process so that the Gaussian noise SD can be estimated without human input and that the results obtained can be reproduced. However, the most critical problem facing current automatic methods is the separation of pure noise from noisy signals and other artifacts because the way in which the automatic methods of Sijbers et al. [22] and of Chang et al. [21] work is by lumping the values of all the pixels from an entire image or from an entire volumetric data set into a one-dimensional array and then estimating the Gaussian noise SD from the histogram of this one-dimensional array. Complicated criteria and techniques have been developed by Sijbers et al. [22] and Chang et al. [21] to separate pure noise from noisy signals and other artifacts from the histogram alone. In this work, we introduce a simpler paradigm for performing noise assessment in MRI. The proposed method shows improved performance compared to previous methods, and may have application in other scientific and technological areas as well.

One of the major aims of this paradigm is to help us get out of the "*one-dimensional*" predicament faced by the automatic methods of Sijbers et al. and Chang et al. so that the separation of pure noise from noisy signals and other artifacts can be done more cleanly and simply. A moment of reflection will indicate that the identification of noise-only pixels should be a part of the paradigm in order to enhance the performance and accuracy of the estimation process, but the identification of noise-only pixels entails some *a priori* knowledge of the Gaussian noise SD. Therefore, any

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paradigm that attempts to make the identification of noise-only pixels a part of the overall noise assessment protocol will necessarily be iterative. Such a paradigm, if feasible, not only can improve the accuracy of the estimate of the Gaussian noise SD but also can provide spatial distributions of noise for further analysis or for quality control and calibration.

In this work, we will present one such paradigm. We will demonstrate that (a) the identification of noise-only pixels, which has not received much attention in MRI literature, is as important as—if not more important than—the estimation of Gaussian noise SD, (b) the identification of noise-only pixels and the estimation of Gaussian noise SD via the sample median (or the sample mean or other optimal sample quantiles, see Appendix B) can be combined into a single coherent framework of noise assessment, and (c) this framework can be made self-consistent, that is, it can be turned into a fixed-point iterative procedure.

Briefly, we propose a novel approach to simultaneously identify noise-only pixels and estimate the Gaussian noise SD from a commonly used data structure (see Fig. 1) in MRI. The data structure as shown in Fig. 1 is ubiquitous in functional MRI and diffusion MRI. It is composed of a series of images acquired at the same physical (slice) location but not necessarily acquired under the same experimentally controlled conditions. Hereafter, we shall refer to the proposed technique as PIESNO, which stands for Probabilistic Identification and Estimation of Noise. PIESNO consists of two distinct parts that are connected dynamically in an iterative manner. The first part of PIESNO is the proposed probabilistic technique for identifying noise-only pixels, which is specifically formulated to deal with the data structure mentioned above. Here, it is assumed that the noise variance is uniform both within and across images in this data structure. The second part of PIESNO is the estimation of Gaussian noise SD via the sample median (or the sample mean or other optimal sample quantiles, see Appendix B), which will be outlined below.

The proposed probabilistic identification of noise-only pixels is designed to take advantage of this data structure to increase the discriminative power to identify noise-only pixels. Specifically, it identifies noise-only pixels through the distribution of the mean of a collection of measurements, shown as a vertical column of data along the k axis in Fig. 1. Consequently, the discriminative power of the identification, which is related to the sharpness of the distribution of the mean, increases as the number of images within the proposed data structure increases. For completeness, we will show that the distribution of the mean used in this work is a Gamma distribution, which is well-known in MRI, e.g., see [24].

Our technique for estimating Gaussian noise SD is based on the median method but we also provide other methods of estimation based on the sample mean and optimal sample quantiles. The median method is a simple formula of the Gaussian noise SD expressed in terms of the sample median of a collection of noise-only measurements. We chose the sample median for its ease of use. The theoretical reason behind our choice of the sample median over other slightly more optimal methods based on the sample quantile of a specific order is explicated in Appendix B.

Both the identification of noise-only pixels and the estimation of noise SD are integral parts of the proposed framework on noise assessment because they are connected dynamically and iteratively to identify noise and estimate noise SD in a self-consistent manner.

Both experimental and simulated data were used to investigate the feasibility, accuracy, stability and global property of the proposed framework. Our approach managed to tease apart two noise distributions through a simple global analysis based on a well-know graphical technique in nonlinear dynamics known as *Cobweb* [25].

A comparison between our technique and Sijbers' [22] (hereafter referred to as the Sijbers Method) was performed. Our technique demonstrated a lower mean squared error in estimating the Gaussian noise SD and a combined method based on both our technique and the Sijbers Method was found to be the most optimal when the number of images within the data structure was above five.

2. Methods

2.1. Theoretical background

In this section, we will first provide the necessary details about the distribution of the arithmetic mean of K independent Gamma random variables, which is also a Gamma distribution, and then establish the connection between the proposed data structure and this well-known distribution by a few simple changes of variables. Throughout this section, we will use the similar notation as employed in [26].

It is known that magnitude MR signals, m (or $m_{i,j,k}$ in Fig. 1), reconstructed from the sum-of-squares algorithm through an N-receiver-coil MRI system [27] follow a nonCentral Chi, $\tilde{\chi} \equiv m/\sigma_g$, distribution of $2N$ degrees of freedom with the non-centrality parameter given by η^2/σ_g^2. The probability density function (PDF) of $\tilde{\chi}$ is given by [23,26]:

$$p_{\tilde{\chi}}(m|\eta,\sigma_g,N) = \frac{m^N}{\sigma_g^2 \eta^{N-1}} \exp\left(-\frac{m^2+\eta^2}{2\sigma_g^2}\right) I_{N-1}\left(\frac{m\eta}{\sigma_g^2}\right), m \geqslant 0 \quad (1)$$

where the PDF is zero for $m < 0$, η is the underlying (combined) signal intensity, σ_g is the Gaussian noise SD, and I_k is the kth-order

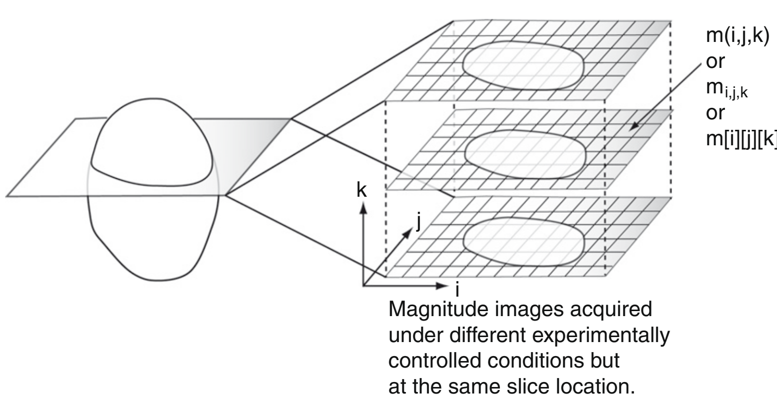

Fig. 1. The proposed data structure includes volumetric data composed of magnitude MR images that are acquired at the same slice location but not necessarily under identical experimentally controlled conditions.