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Design of self-refocused pulses under short relaxation times

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ABSTRACT

The effect of using self-refocused RF pulses of comparable duration to relaxation times is studied in detail using numerical simulation. Transverse magnetization decay caused by short T2 and longitudinal component distortion due to short T1 are consistent with other studies. In order to design new pulses to combat short T1 and T2 the relaxation terms are directly inserted into the Bloch equations. These equations are inverted by searching the RF solution space using simulated annealing global optimization technique. A new T2-decay efficient excitation pulse is created (SDETR: *s*ingle *d*elayed *e*xcursion *T*2 *r*esistive) which is also energy efficient. Inversion pulses which improve the inverted magnetization profile and achieve better suppression of the remaining transverse magnetization are also created even when both T1 and T2 are short. This is achieved, however, on the expense of a more complex B1 shape of larger energy content.

1. Introduction

When imaging samples with very short T2 (e.g. bone, tissue injected with contrast agent, or porous media) it is desirable to start acquiring signal immediately after the end of the RF excitation pulse. This has been achieved using self-refocused pulses which remove the need for applying gradient reversal usually needed for rewinding spins' accumulated linear phase. The family BURP (*b*and-selective *u*niform-*r*esponse *p*ure-phase) RF pulses [1] have been designed using optimization techniques. Although these pulses shorten total excitation time spins still relax during the excitation leading to reduced NMR signal. In many cases the design criteria have ignored relaxation processes by assuming that both relaxation times T1 and T2 are much longer than the RF pulse duration (L). While this assumption is correct in many cases especially when short RF pulses (known as hard pulses) are used to excite bulk samples, however, there are applications in which relaxation times have to be taken into account when they become comparable to L. For example T1 is shortened considerably in T1-weighted dynamic contrast-enhanced imaging and also when paramagnetic impurities and surface relaxation make T1 comparable to L in porous media imaging.

Many studies have analyzed the effect of short relaxation times (in particular T2) on the NMR signal [2–6]. This was done by inserting already known RF profiles (e.g. Burp, Gaussian, sinc, SNOB, etc.) into the *forward* calculation of the Bloch equations using numerical techniques. Others have attempted to take into account the effects of relaxation into the design of the RF pulse [3,7–9]. This is usually a more demanding problem since it involves *inverting* the Bloch equations and the design will be tailored to specific value of T1 and T2, and therefore may not be optimum for exciting all values of T1 and T2.

Results obtained here confirm most of the studies referenced in this work that short T2 attenuate magnetization in the selected slice region while short T1 produce significant distortions in the profiles. We extend this work by seeking new slice-refocused RF profiles that reduce the effects of short relaxation times.

2. Theory

We use simulated annealing (SA) global optimization techniques [1,10–12] to synthesize self-refocused RF pulses (B1 profiles). Typically the annealing procedure relies on gradually reducing an initial value (T_0) of a control parameter (temperature analog in thermodynamics) by a small amount (temperature step ΔT). At each temperature a series (N) of random search processes are performed to search for the minimum of an error function at certain system configuration. The system variables are the set of desired B1 values. The error is the mean square difference between the target response and the excited magnetization. The latter is calculated by numerically solving the Bloch equations, using Runge–Kutta–Fehlberg 6th order method [13]. In this work, and unlike previously published reports on SA, we include the relaxation terms in the Bloch equations to investigate the RF profiles under conditions when T1 and T2 are comparable to L. The Bloch equations then become as in Eq. (1)

$$\frac{\partial \vec{M}}{\partial t} = \gamma(\vec{M} \times \vec{B}) - R\vec{M} + R1\vec{M}_0 \quad (1)$$

where $\vec{M} = (M_x, M_y, M_z)^T$ is the magnetization column vector, $\vec{B}$ is the effective RF field in the laboratory frame, and γ is the gyromag-

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netic ratio. R is a 3×3 diagonal matrix with elements $R2$, $R2$, and $R1$, and $\vec{M}_0 = (0, 0, M_0)^T$ is the thermal equilibrium magnetization column vector.

The frequency response is discretized into M elements covering the slice selection direction for the three magnetization components (M_{ij}, $i = 1, 2, 3$ for the x, y, and z components respectively, and $j = 1$ to M for the space elements). The target magnetization profile (M_{ij}^{target}, with similar meaning for the indices i and j as before) is the desired magnetization response due to the application of the RF pulse. The RF pulse can be, for example, a $\pi/2$ excitation pulse or a π inversion pulse. The objective (or error) function (Eq. (2)) that we seek to minimize is

$$objective = \sum_{i=1}^{3} \sum_{j=1}^{M} \left(M_{ij}^{\text{target}} - M_{ij} \right)^2 \tag{2}$$

Constraints on the maximum and minimum permissible values of the magnetization components (M_{ij}^{upper} and M_{ij}^{lower}) are also defined. The number of the constraints along each axis can be smaller than the maximum M if some regions of the frequency response are left unconstrained for some or all of the magnetization components. For example, the transition region between the excited slice and suppression (unexcited) regions is unconstrained for all magnetization components, while the M_z component is left free in the excited slice region for the excitation pulse [1]. For this work we have constrained all magnetization components according to Eq. (3) to design a $\pi/2$ excitation pulse

$$\left.\begin{array}{ll} -\varepsilon_{lower} < M_x < \varepsilon_{upper}; & \text{for all } v \\ M_0 - \varepsilon_{lower} < M_y < M_0 & \text{for } |v| < \Delta F/2 \\ -\varepsilon_{lower} < M_y < \varepsilon_{upper}; & \text{for } |v| > \Delta F/2 + \delta F \\ -\varepsilon_{lower} < M_z < \varepsilon_{upper}; & \text{for } |v| < \Delta F/2 \\ M_0 - \varepsilon_{lower} < M_z < M_0; & \text{for } |v| > \Delta F/2 + \delta F \end{array}\right\} \tag{3}$$

M_0 is set to unity and $\varepsilon_{\text{lower}}$ and $\varepsilon_{\text{upper}}$ are the tolerance values (set to 0.001). ΔF is the slice thickness and δF is the transition region width, both in Hz.

In order to speed up the optimization process the constrained problem is transformed into an unconstrained one. This is achieved by inserting the constraints into the error function using the method of penalties [13,14]. The contribution of the constraints is controlled through the weighting or emphasis factors α_i, $i = 1, 2, 3$ for the x, y, and z components, respectively (chosen for the results presented here to be equal to each other). The objective function then becomes

$$objective = \sum_{i=1}^{3} \left\{ \sum_{j=1}^{M} \left(M_{ij}^{target} - M_{ij} \right)^2 + \alpha_i \left(\sum_{j=1}^{M} \left(M_{ij}^{upper} - M_{ij} \right)^2 + \sum_{j=1}^{M} \left(M_{ij}^{lower} - M_{ij} \right)^2 \right) \right\} \tag{4}$$

L is chosen to be 3.0 ms and modeled discretely by 30 ordinates and the maximum number of constraints M is set to 64 in this work. Searching the error surface for the minimum error is accomplished by varying the B1 profile. This is implemented indirectly through the use of Fourier series coefficients [1,11] with six harmonics yielding a total of 13 adjustable coefficients. This procedure ensures continuous and smooth RF profile [15] and reduces the computational task by searching through a smaller dimensional space (i.e. 13 instead of 30). The B1 waveform is given by Eq. (5)

$$\mathrm{B1}(t_i)P = a_0 + \sum_{n=1}^{6} a_n \cos(n w_0 t_i) + b_n \sin(n w_0 t_i) \qquad i = 1, 2, \ldots, 30 \tag{5}$$

$w_0 = 2\pi/L$ is the fundamental frequency.

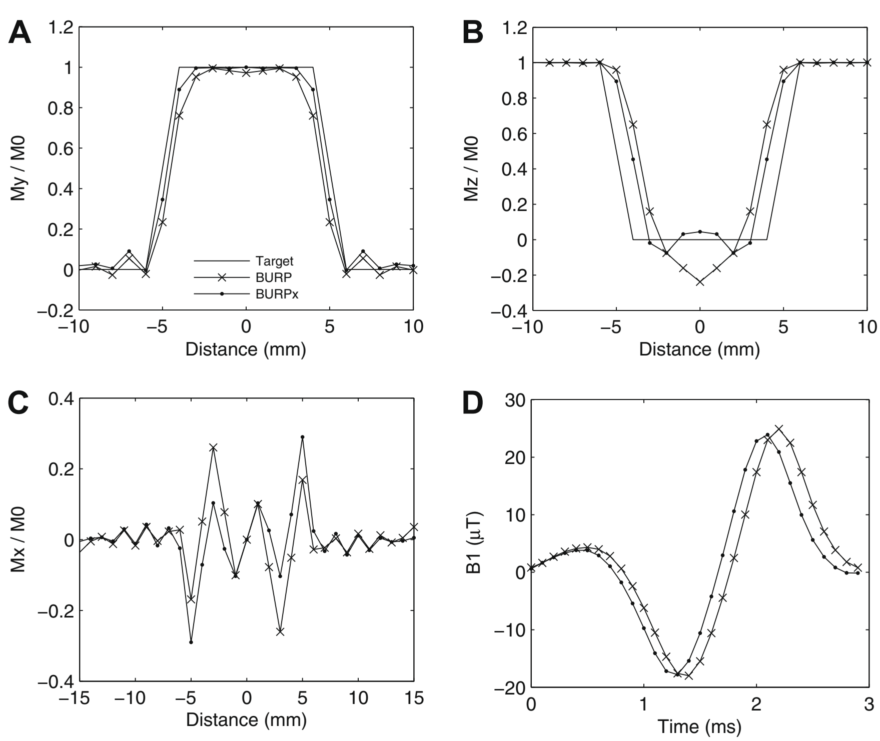

Fig. 1. A $\pi/2$ excitation pulse is optimized using constraints applied to all magnetization components over all frequency regions (extended constraints BURPx) as opposed to the BURP pulses (which have no constraints over the z-component). Excited magnetization component is improved slightly (A and B). Furthermore, the BURPx profile (D) appears slightly shorter than BURP and therefore will be adopted as a standard for comparison with other designed pulses in this work. Similar out of phase transverse magnetization (C) are produced by the two pulses.