Journal of Magnetic Resonance 198 (2009) 49–56

Contents lists available at ScienceDirect

Journal of Magnetic Resonance

journal homepage: www.elsevier.com/locate/jmr

Optimal decay rate constant estimates from phased array data utilizing joint Bayesian analysis

James D. Quirk [a,*], Alexander L. Sukstanskii [a], G. Larry Bretthorst [a], Dmitriy A. Yablonskiy [a,b]

[a] Mallinckrodt Institute of Radiology, Washington University School of Medicine, 4525 Scott Avenue, Campus Box 8227, St. Louis, MO 63110, USA
[b] Department of Physics, Washington University, St. Louis, MO, USA

ARTICLE INFO

Article history:
Received 16 October 2008
Revised 27 December 2008
Available online 13 January 2009

Keywords:
Magnetic resonance
Magnetic resonance imaging
Bayesian probability theory
Phased array coil
Exponential parameter estimation

ABSTRACT

Since their initial description, phased array coils have become increasingly popular due to their ease of customization for various applications. Numerous methods for combining data from individual channels have been proposed that attempt to optimize the SNR of the resultant images. One issue that has received comparatively little attention is how to apply these combination techniques to a series of images obtained from phased array coils that are then analyzed to produce quantitative estimates of tissue parameters. Herein, instead of the typical goal of maximizing the SNR in a single image, we are interested in maximizing the accuracy and precision of parameter estimates that are obtained from a series of such images. Our results demonstrate that a joint Bayesian analysis offers a "worry free" method for obtaining optimal parameter estimates from data generated by multiple coils (channels) from a single object (source). We also compare the properties of common channel combination techniques under different conditions to the results obtained from the joint Bayesian analysis. If the noise variance is constant for all channels, a sensitivity weighted average provides parameter estimates equivalent to the joint analysis. If both the noise variance and signal intensity are similar in all channels, a simple channel average gives an adequate result. However, if the noise variance differs between channels, an "ideal weighted" approach should be applied, where data are combined after weighting by the channel amplitude divided by the noise variance. Only this "ideal weighting" provides results similar to the automatic-weighting inherent in the joint Bayesian approach.

1. Introduction

Since their early description [1], phased array coils have become increasingly widespread due to their ease of customization for various applications. Their recent surge in popularity can be traced to improvements in coil technologies and the development of rapid imaging techniques that utilize the spatial information from the phased array coils to decrease acquisition times [2,3]. However, these coils are also used in more traditional imaging experiments simply for their flexibility and increased signal-to-noise ratio (SNR).

Numerous investigators [1,4–9] have attempted to optimize the SNR of images from phased array coils and proposed various techniques for combining such data. Roemer [1,5] and others [6,7,9] have suggested that the sum-of-squares (SOS) combination provides a near optimal signal-to-noise ratio in the reconstructed image, approaching that of a reconstruction using the "correct" channel sensitivity profile without requiring additional acquisitions. Others have refined this technique by weighting the channels using more complex factors that reduce the impact of local signal and noise fluctuations and more accurately characterize the true coil response profiles [6,8,10–12]. These weighting factors are commonly obtained from either a smoothed version of the data itself or a separately acquired lower-resolution image.

One issue that has received relatively little attention is how the different channel combination techniques affect parameter estimates obtained from modeling the signal in a series of images [11,13]. Here, we are interested in maximizing the accuracy and precision of parameter estimates that are obtained from a series of array coil images. In addition to SNR optimization, this imposes the additional constraint of accurately preserving the relationships between the series images.

We also explore the effects of various channel combination methods on the accuracy and precision of parameter estimates and examine the case where the common assumption of equal noise power across channels is violated. In the simplest method for combining channels, the channel average or sum, all channels are treated equally. When signals of differing SNR are averaged, information from the high SNR channels are diluted by the lower SNR channels, resulting in less accurate and less precise parameter estimates [14]. Such variations in signal and noise power across channels are common in real imaging experiments as the array ele-

* Corresponding author. Fax: +1 314 362 0526.
E-mail address: jdquirk@wustl.edu (J.D. Quirk).

1090-7807/$ - see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.jmr.2009.01.001

ments are seldom equidistant from a particular region of interest. If there are systematic effects in the data that are not properly modeled (e.g. the Rican noise profile induced by processing the magnitude images from each channel), they can coherently combine and further distort the parameter estimates. Channels may also experience different loading due to their placement on different parts of the sample or patient, and will have some degree of coupling. Thus, the simple averaging of channels is rarely advisable.

Sensitivity-weighted averaging attempts to mitigate these effects by accounting for spatial variations in signal intensity for each channel. However, these methods do not account for variations in noise power between channels and can still magnify systematic effects and artifacts in the data. If the channel weighting factors for each image in a series is derived from its own intensity, these weighting factors will differ across the series images, potentially biasing the parameter estimates. An extreme case of this occurs with the sum-of-squares (SOS) combination of images. While reported to provide a "near-optimal" SNR, the SOS combination also artificially distorts the relationships between images in a series by forcing all low SNR points to take positive values, introducing a DC offset that coherently combines across channels. For an exponential decay model, this increased noise floor produces a systematic underestimation of the decay rate constant and will affect the accuracy and precision of rate constant estimation, as previously described [6,15–20].

As an alternative to determining the optimal channel weighting factors for a given experimental setup, we could also analyze multi-channel data without signal combination. The signals from the various channels can be jointly analyzed with a model that allows the channel-specific properties (such as signal amplitude and noise power) to vary across channels while requiring the MRI properties inherent to the imaged object (such as a decay rate constant) to be identical for all channels. We have implemented this framework using Bayesian probability theory and demonstrate its benefits for modeling simulated multi-channel data compared to more traditional combination techniques. For simplicity, we consider here only the mono-exponential decay model prevalent in MR, but these general principles have an obvious extension to more complex estimation problems. We conclude that a joint Bayesian analysis offers a "worry free" method for obtaining optimal parameter estimates from multi-channel data.

2. Theory

A ubiquitous model in MRI experiments is the mono-exponential decay. For simplicity of the analysis below, we assume a simple mono-exponential analysis without a constant, such as in T_2 or diffusion measurements. For an array of M-channels used to acquire decay measurements at N different times, the measured signal can be expressed as:

$$S_m(t_n) = A_m \exp(-R\, t_n) + \eta_m(t_n), \qquad (1)$$

where $S_m(t_n)$ is the signal measured on the mth channel at the nth sampling time (or b-value in the case of the diffusion experiment), $\eta_m(t_n)$ is the noise in the mth channel at the nth sampling time t_n, A_m is the signal amplitude in channel m, and R is a rate constant (e.g. R_2 or ADC). While we assume for simplicity that the channels are sampled simultaneously, which is typically the case, and that there is no coupling between coils, no other assumptions are made as to the distribution of data samples in time. The rate constant is treated as an inherent property of the sample, and is therefore independent of which channel is performing the measurement, whereas the signal amplitude and the noise are properties of each channel.

NMR/MRI scanners typically produce a complex signal (quadrature detection), and the real and imaginary components are commonly combined to produce a magnitude signal or image. For our purposes, the complex signal from each channel is assumed to have been "phased", i.e. the coherent signal moved entirely to the real channel, and only the real signals are analyzed [21,22]. This produces an improvement in SNR and removes the bias introduced by using magnitude images. An alternative would be the simultaneous analysis of the real and imaginary components from all channels. However, as this would complicate the model by introducing an additional amplitude (or phase) for each channel, we will assume for simplicity that the data have already been phased.

In Bayesian analysis, we are interested in calculating $p(AR|D\sigma I)$, the joint posterior probability of the model parameters A and R given the data, D, the standard deviation of the noise prior probability, σ, and the prior information, I. Using Bayes' theorem and the product rule, omitting constant terms that will cancel upon normalization, and assuming independence in our prior knowledge of A, R, and σ, this can be expressed as

$$p(AR|D\sigma I) \propto p(R|I)p(A|I)p(D|AR\sigma I). \qquad (2)$$

In this equation, $p(A|I)$ and $p(R|I)$ are the prior probabilities of the parameters A and R, and represent what is known about the possible values of these parameters before acquiring the data; $p(D|AR\sigma I)$ is the direct probability of the data given the parameters and is proportional to a likelihood function.

Initially, we consider the signal generated by a single channel and calculate the expected uncertainty in the resultant parameter estimates. Using uniform and comparatively non-informative priors, the joint posterior probability of the model parameters in Eq. (2) can be expressed as [14,23,24]

$$p(AR|D\sigma I) \propto \exp\left(-\frac{Q}{2\sigma^2}\right); \quad Q = \sum_{n=0}^{N-1} [D(t_n) - A\exp(-Rt_n)]^2. \qquad (3)$$

In the majority of exponential decay experiments, the actual value of the amplitude parameter is of little interest and we are primarily concerned with estimation of the rate constants. In such cases, the amplitudes can be removed from the analysis by calculating the marginal probability for the decay rate constant, R. This requires integrating Eq. (3) over all possible values of A:

$$p(R|D\sigma I) \propto \int dA\, p(AR|D\sigma I) \propto \int dA\, \exp\left(-\frac{Q}{2\sigma^2}\right). \qquad (4)$$

Assuming high SNR, that the data are sampled at uniformly spaced times, and that the data are acquired until the exponential decays into the noise, the uncertainty in the decay rate constant estimate for a single channel was previously estimated as the standard deviation of the posterior probability distribution for parameter R, $\sqrt{8\hat{R}^3\Delta t}/\mathrm{SNR}$, where Δt is the sampling interval between data points and $\hat{R}$ is the true value of the rate constant [25].

To broaden the applicability of this result to imaging experiments, here we relax the assumptions of uniform sampling and acquiring data until the signal decays into the noise. In the next section, we will also expand this result to the multi-channel case. To analytically evaluate Q in Eq. (4), the data are expressed in terms of $\hat{A}$, $\hat{R}$, and $\hat{\eta}(t_n)$, the true values of the amplitude, rate constant, and noise:

$$D(t_n) = \hat{A}\exp(-\hat{R}t_n) + \hat{\eta}(t_n). \qquad (5)$$

In the high SNR approximation, which allows us to neglect the noise term from Eq. (5), evaluation of the amplitude integral in Eq. (4) and omitting constant terms produces the marginal probability for the rate constant in the form:

$$p(R|D\sigma I) \propto \exp\left(\frac{(\mathbf{d}\cdot\mathbf{G})^2}{2\sigma^2(\mathbf{G}\cdot\mathbf{G})}\right), \qquad (6)$$