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Communication

Scaling laws for transverse relaxation times

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Abstract

Simple scaling laws are useful tools in understanding the effect of changing parameters in MRI experiments. In this paper the general scaling behavior of the transverse relaxation times is discussed. We consider the dephasing of spins diffusing around a field inhomogeneity inside a voxel. The strong collision approximation is used to describe the diffusion process. The obtained scaling laws are valid over the whole dynamic range from motional narrowing to static dephasing. The dependence of the relaxation times on the external magnetic field, diffusion coefficients of the surrounding medium, and the characteristic scale of the field inhomogeneity is analyzed. For illustration the generally valid scaling laws are applied to the special case of a capillary, usually used as a model of the myocardial BOLD effect.

Keywords: Transverse relaxation; Susceptibility effects; Diffusion; Scaling

1. Introduction

The transverse relaxation times T_2 and T_2^* are fundamental quantities in MRI, especially for characterizing tissues and their properties. The influence of static magnetic field inhomogeneities on relaxation times is of special interest in understanding relaxation processes inside the voxel. It is well known that susceptibility contrasts, external magnetic fields and diffusion influences the relaxation times T_2 and T_2^*. Obviously it is useful to understand the scaling of relaxation times with respect to their parameters in order to describe effects of parameter changing in a simple way. For example scaling laws can be used to predict the effects of changing external magnetic field strength or concentration of contrast agents. It is important to know how relaxation times vary subjected to changes in these characteristic quantities. Despite this fact papers dealing with this issue are sparse in the literature. Weisskoff et al. [1] discussed the scaling behavior of the relaxation rate R_2 in the context of microscopic susceptibility variations. Starting from the Bloch-Torrey-Equation [2] they obtained two special scaling laws and verified them by extensive numerical simulation. Employing the strong collision approximation [3–5] we give a rigorous derivation of a generalized scaling law for both transverse relaxation times T_2 and T_2^*. After considering the basic model of a field inhomogeneity inside a voxel we use well known results of Bauer et al. [3] in order to obtain the scaling laws. To give an example these results are applied to a cylindrical geometry which is commonly used as a model of a vascular network.

2. Basic model

We consider an arbitrary distribution of magnetic material G inside a voxel causing a susceptibility shift $\Delta\chi = \chi_i - \chi_e$ compared to the surrounding medium with volume V (see Fig. 1). The volume fraction η of material inside the voxel is given by $\eta = G/(G+V)$. Dephasing takes place in the remaining volume V of the voxel around the magnetic perturber G, in which the diffusion of the spins is determined by the diffusion coefficient D.

The z-component of the magnetic field caused by the inhomogeneity G is given by [6,7]

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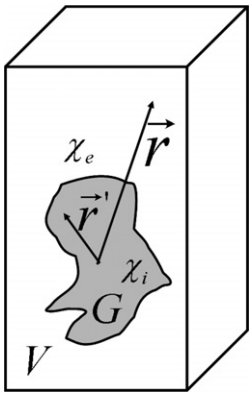

Fig. 1. Voxel containing a magnetic inhomogeneity G with susceptibility χ_i and dephasing volume V with susceptibility χ_e. All coordinates inside G are represented by primed vectors $\mathbf{r}'$ and all coordinates of V by unprimed vectors $\mathbf{r}$.

$$B_z(\mathbf{r}) = B_0 \Delta\chi \frac{\partial^2}{\partial z^2} \int_G \frac{\mathrm{d}^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}, \tag{1}$$

where $\mathbf{r}'$ represents the coordinates inside the volume of the perturber G and $\mathbf{r}$ the coordinates of the surrounding volume V (see Fig. 1). This means that only the field from the magnetic inhomogeneity G inside the voxel influences the dephasing of spins in V and the effects of neighboring voxels are neglected. The external magnetic field B_0 induces the local resonance frequency $\omega(\mathbf{r}) = \gamma B_z(\mathbf{r})$ which can be written as

$$\omega(\mathbf{r}) = \delta\omega f(\mathbf{r}), \tag{2}$$

where the characteristic frequency is given by

$$\delta\omega = \gamma\Delta\chi B_0, \tag{3}$$

which contains the susceptibility properties of the perturber and the external magnetic field, while the geometric function

$$f(\mathbf{r}) = \frac{\partial^2}{\partial z^2} \int_G \frac{\mathrm{d}^3\mathbf{r}\prime}{|\mathbf{r}-\mathbf{r}'|} \tag{4}$$

defines the shape of the perturber. Thus we are able to separate the susceptibility properties of the inhomogeneity G from its distribution inside the voxel. This offers the possibility to investigate the influence of each part independently.

In order to derive the scaling laws some independent approximations are necessary. First we obtain the correlation time τ by using a mean time approximation that considers a monoexponential time behavior of the correlation function. The second one is the strong collision approximation which is used to simplify the diffusion process around field inhomogeneities. The last approximation is again a mean time approximation which considers the magnetization decay as monoexponential also. As shown in previous work [8] a relation between the correlation function and the magnetization decay exists in the gaussian approximation. As it is not known if the considerations of it are fullfiled generally [9] we use the strong collision approach to describe the magnetization decay.

We start with calculating the correlation time τ which is required to obtain the magnetization decay. To investigate the dynamic property of the problem we use the two-point correlation function of the stochastic field fluctuations to which a spin is subjected. It is defined as

$$K(t) = \int_V \mathrm{d}^3\mathbf{r} \int_V \mathrm{d}^3\mathbf{r}_0\, \omega(\mathbf{r}) p(\mathbf{r},\mathbf{r}_0,t) p(\mathbf{r}_0)\omega(\mathbf{r}_0), \tag{5}$$

where $p(\mathbf{r}, \mathbf{r}_0, t)$ is the probability density of finding a spin at point $\mathbf{r}$ after time t with the initial ($t = 0$) position $\mathbf{r}_0$, and $p(\mathbf{r}_0)$ specifies the probability density function of the equilibrium distribution. In our case the latter is identical with the spin density, which we assume to be homogeneous, i.e. $p(\mathbf{r}_0) = 1/V$. Assuming free diffusion of spins within V, the probability $p(\mathbf{r},\mathbf{r}_0,t)$ is simply the Green's function of the diffusion equation where D is the diffusion coefficient

$$\frac{\partial}{\partial t} p(\mathbf{r},\mathbf{r}_0,t) = D\nabla^2 p(\mathbf{r},\mathbf{r}_0,t) \tag{6}$$

or

$$p(\mathbf{r},\mathbf{r}_0,t) = \mathrm{e}^{tD\nabla^2}\delta(\mathbf{r}-\mathbf{r}_0), \tag{7}$$

with the reflectory boundary conditions $\partial_r p(\mathbf{r},\mathbf{r}_0,t) = 0$ at the surface of the magnetic perturber and the voxel boundaries. In the case of permeable membranes the probability function $p(\mathbf{r},\mathbf{r}_0,t)$ has to fulfill the radiation boundary conditions $\partial_r p(\mathbf{r},\mathbf{r}_0,t) = kp(\mathbf{r},\mathbf{r}_0,t)$ at the surfaces of the inhomogeneities where k is the permeability of the membrane. Insertion of the probability density Eq. (7) and $p(\mathbf{r}_0)$ into the definition of the correlation function Eq. (5), results in

$$K(t) = \frac{1}{V}\int_V \mathrm{d}^3\mathbf{r}\omega(\mathbf{r})\mathrm{e}^{tD\nabla^2}\omega(\mathbf{r}). \tag{8}$$

Using Eq. (2) the correlation function at $t = 0$ is given by

$$K(0) = \frac{1}{V}\int_V \mathrm{d}^3\mathbf{r}\omega^2(\mathbf{r}) =: \langle\omega^2(\mathbf{r})\rangle = \delta\omega^2\langle f^2(\mathbf{r})\rangle. \tag{9}$$

The result $K(0) \sim \delta\omega^2$ is the same as from Jensen and Chandra [10], Eq. (18) and in complete agreement with the more general Eq. (1) given by Sukstanskii and Yablonskiy [11]. In general the correlation function $K(t)$ does not exhibit a single exponential decay as is often assumed [12]. This hampers a simple determination of the correlation time, i.e. $K(t) \sim \mathrm{e}^{-t/\tau}$. However, a proper definition of the correlation time is to define it as the mean relaxation time of the correlation function, i.e. according to ref. [13],

$$\tau = \int_0^\infty \mathrm{d}t\, \frac{K(t)}{K(0)}. \tag{10}$$

In fact, it has been demonstrated that this definition provides the best single exponential approximation of the correlation function. A commonly used approximation for the correlation function is