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Mixed convective peristaltic flow of Sisko fluid in curved channel with homogeneous-heterogeneous reaction effects

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ABSTRACT

The principal emphasis of this article relates the simultaneous effects of shear thinning and shear thickening in mixed convective peristaltic flow of Sisko fluid. The channel boundaries are considered curved in shape with compliant properties. The severity of gravitational effects are retained in the flow along with viscous heating and thermal radiation. In addition homogeneous-heterogeneous reactions effects are also examined. Thus mathematical description of above quantities results in nonlinear ODEs. The simplified system is then addressed numerically. The results signify the pronounced behavior of velocity and temperature corresponding to shear thinning. Concentration bears opposite response for homogeneous and heterogeneous parameters.

1. Introduction

The theory of non-Newtonian liquids has been established since Navier–Stokes relations are found inadequate to describe the rheological complex fluids like petroleum, blood, shampoos, greases, muds, oils, paints, lubricants, polymer solutions etc. Non-Newtonian fluids comprised nonlinear relation between shear stress and strain rate. Until now different classical models have been proposed to explore non-Newtonian relationships depending on the rheological properties. In lubricants for instance, a power law model is used in which both dilatant and pseudoplastic behavior are addressed. Thus among these liquids, Sisko fluid model as a generalized version is accomplished with shear thinning and shear thickening attributes. Further with suitable selection of material fluid parameters Sisko fluid can predict many typical properties of Newtonian and non-Newtonian liquids. Some relevant studies in this direction can be consulted via refs. [1–5].

The peristalsis is periodic propulsion of fluid due to advancing waves along the channel through involuntary contractions and relaxations. Such movements are of prime importance in several operations involved in physiological and industrial processes. In addition to involvement of peristalsis in food digestion, lymph transport, urine passage, finger, roller, waste management and hose pumps, the peristalsis found its applications in designing some biomedical instruments such as heart lung machine and blood pumps. In spite of wide range utility of peristaltic phenomenon the theoretical research on the topic was commenced by Latham [6] and Shapiro et al. [7]. Afterwards peristalsis is shown centre of interest for many researchers to date (see refs. [8–12]). In reference to blood arterial walls and capillaries the peristalsis of curved channel is practically important. Thus a mathematical study relative to peristalsis through curved geometry is considered more accurate. However the topic is not much studied in the literature (see refs. [13–16]). It is further added that consideration of compliant wall properties is significantly important in blood flow regimes, urethras walls and flow of air in lungs. Thus many researchers put forward their efforts to analyze compliant characteristics of walls in curved flow peristalsis (see refs. [17–19]).

Based on the physical state (i.e., color, shape, length, size, weight, distribution, appearance, language, income, disease, temperature, radioactivity, architectural pattern, etc.) the materials in chemical reaction are characterized through homogeneous and heterogeneous reactions. The former occurs in a single phase (gaseous, liquid, or solid) whereas the later as components of two or more phases. Homogeneous reactions are theoretically simple when compared with heterogeneous reactions since reacting product depends only on the nature of reacting species. On the other hand heterogeneous reactions preserve practical importance as it relates

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dependence of product on nature of two or more different reacting species. Such reactions are witnessed in batteries, corrosion phenomenon and electrolytic cells. Also certain chemically reacting processes comprised homogeneous and heterogeneous reactions. In such cases catalyst (agent) is used to accelerate the reaction speed so that reaction continued to the desired limit. At present reasonable amount of literature is available on flows with homogeneous-heterogeneous reaction (see refs. [20–24]). However scarce information on peristalsis with homogeneous and heterogeneous reactions is noticed in the refs. [25–27].

Mixed convection arises when gravitational effects are strong enough to promote heat transfer. Mixed convection flows along with thermal radiation plays a significant role in overall surface heat transfer in situations where heat transfer by only convection means is inadequate. The combined effects of mixed convection and thermal radiation is of fundamental importance in organs of human physiology especially in liver, brain, heart and in contraction of skeletal muscles. Radiation can control the excess heat generation inside the body since high temperatures pose serious stresses for the human body placing it in serious danger of injury or even death. In varying climate conditions the skin vasodilatation and sweating are maintained by mixed convection and radiation. When skin temperature is greater than that of the surroundings the body can lose heat by radiation and conduction. Whereas when the temperature of the surroundings is greater than that of the skin the body actually gains heat by radiation and conduction to keep body at healthy level. Having all such in mind the representative studies on mixed convection/radiation of nonlinear fluids are given in the attempts [28–40].

Motivated by the aforementioned utility of various effects, the purpose here is twofold. Firstly to explore and compare the shear thinning and thickening effects corresponding to mixed convection, thermal radiation and viscous heating in peristaltic flow of an incompressible Sisko fluid. Secondly to inspect homogeneous-heterogeneous reactions effects. It is remarkable to notice that such reactions are involved in velocity via mixed convection. To the best of authors knowledge such attempt has not been reported to date. Hence the flow governing equations have been modeled and solution expressions are approximated for the graphical results. The physical interpretation has been made in the last section.

2. Problem formulation

Mathematical formulation for an incompressible Sisko fluid (comprising shear thinning and shear thickening effects) bounded in a curved channel of inner radius R^* and separation $2\bar{d}$ has been modeled in this section. The dynamics of flow geometry is configured in such a way that peristaltic wave while propagating along the channel boundaries in radial direction ($\bar{r}$) generates the flow. Here $\bar{x}$ serves the axial direction (see Fig. 1). In addition the channel geometry is such that the gravitational effects are not ignored. The relevance of compliance in terms of wall's stiffness, elasticity and damping is also present. Further the flow subject to a simple homogeneous and heterogeneous reaction model comprised of two chemical species $\breve{A}$ and $\breve{B}$ having concentration values $\breve{a}$ and $\breve{b}$ respectively is taken in to account.

Thus the following expression gives the relative position of the wall surface in radial direction as sum of separation between channel and sinusoidal peristaltic wave

$$\bar{r} = \pm\bar{\eta}(\bar{x},\bar{t}) = \pm\left[\bar{d} + \bar{\epsilon}\sin\frac{2\pi}{\lambda}(\bar{x} - c\bar{t})\right], \tag{1}$$

where the symbols c, $\bar{\epsilon}$, λ signify the peristaltic wave's speed, amplitude and length, $\bar{t}$ and $\pm\bar{\eta}$ the time and displacements of channel walls. The homogeneous-heterogeneous reaction effects are assumed to have the form [25–27]:

$$\breve{A} + 2\breve{B} \rightarrow 3\breve{B}, \quad rate = k_c\breve{a}\breve{b}^2.$$

However corresponding to catalyst surface the single, isothermal, first order chemical reaction preserves the following mathematical form:

$$\breve{A} \rightarrow \breve{B}, \quad rate = k_s\breve{a}.$$

For the two reactions the same temperature value is assumed where k_c and k_s denote the rate constants.

Following Hayat et al. [27], the mathematical description of the problem relates conservation principles of mass, momentum and energy. Thus in present case one obtains

$$\frac{\partial\bar{v}}{\partial\bar{r}} + \frac{R^*}{\bar{r}+R^*}\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\bar{v}}{\bar{r}+R^*} = 0, \tag{2}$$

$$\rho\left[\frac{d\bar{v}}{d\bar{t}} - \frac{\bar{u}^2}{\bar{r}+R^*}\right] = -\frac{\partial\bar{p}}{\partial\bar{r}} + \frac{1}{\bar{r}+R^*}\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R^*)\bar{S}_{\bar{r}\bar{r}}\right\} + \frac{R^*}{\bar{r}+R^*}\frac{\partial\bar{S}_{\bar{x}\bar{r}}}{\partial\bar{x}} - \frac{\bar{S}_{\bar{x}\bar{x}}}{\bar{r}+R^*}, \tag{3}$$

$$\rho\left[\frac{d\bar{u}}{d\bar{t}} + \frac{\bar{u}\bar{v}}{\bar{r}+R^*}\right] = -\frac{R^*}{\bar{r}+R^*}\frac{\partial\bar{p}}{\partial\bar{x}} + \frac{1}{(\bar{r}+R^*)^2}\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R^*)^2\bar{S}_{\bar{r}\bar{x}}\right\} + \frac{R^*}{\bar{r}+R^*}\frac{\partial\bar{S}_{\bar{x}\bar{x}}}{\partial\bar{x}} + g\rho\beta_T(T-T_0) + \rho g\beta_{\breve{a}}(\breve{a}-\breve{a}_0). \tag{4}$$

$$\rho c_p\left[\frac{dT}{d\bar{t}}\right] = k_1\nabla^2T - \frac{\partial}{\partial\bar{r}}\left(\frac{-16\sigma^*T_0^3}{3l^*}\frac{\partial T}{\partial\bar{r}}\right) + \boldsymbol{\tau}.\quad \text{grad}\quad \mathbf{V}, \tag{5}$$

where the homogeneous-heterogeneous reaction effects can be encountered through the following equations [21,27]:

$$\frac{d\breve{a}}{dt} = D_{\breve{A}}\left(\nabla^2\breve{a}\right) - k_c\breve{a}\breve{b}^2, \tag{6}$$

$$\frac{d\breve{b}}{dt} = D_{\breve{B}}\left(\nabla^2\breve{b}\right) + k_c\breve{a}\breve{b}^2. \tag{7}$$

The Cauchy stress tensor $\boldsymbol{\tau}$ and extra stress tensor $\bar{\mathbf{S}}$ for Sisko fluid model are [1–5]:

$$\boldsymbol{\tau} = -\bar{p}\mathbf{I} + \bar{\mathbf{S}}, \tag{8}$$

$$\bar{\mathbf{S}} = \left[\alpha + \beta\left|\Pi\right|^{n-1}\right]\mathbf{A}_1, \tag{9}$$

where $\mathbf{A}_1 = \text{grad}\mathbf{V} + \text{grad}\mathbf{V}^T$ represents the first Rivlin-Erickson tensor, $\frac{d}{dt} = \frac{\partial}{\partial\bar{t}} + \bar{v}\frac{\partial}{\partial\bar{r}} + \frac{R^*\bar{u}}{\bar{r}+R^*}\frac{\partial}{\partial\bar{x}}$ the material time derivative and $\Pi = \frac{1}{2}tr(\mathbf{A}_1^2)$ the second invariant of symmetric part of velocity gradient. In Sisko fluid model α, β and n (≥ 0) represent the shear rate viscosity, consistency index and power-law index respectively. Involvement of power index n provides an edge to Sisko fluid as shear thinning ($n < 1$), shear thickenning ($n > 1$). Newtonian fluid ($n = 1, \alpha = 0, \beta = \mu$ or $\beta = 0, \alpha = \mu$) behavior can be constituted using this model. Also

$$\nabla^2 = \left(\frac{R^*}{\bar{r}+R^*}\right)^2\frac{\partial^2}{\partial\bar{x}^2} + \frac{1}{\bar{r}+R^*}\frac{\partial}{\partial\bar{r}}\left\{(\bar{r}+R^*)\right\}.$$